2.71/2.710 Optics   **Spring '14**
HW4   Posted Mar. 19, 2014, Due Apr. 02, 2014

1. The **Mach-Zehnder** Interferometer measure the relative phase shift of two separate light paths. It operates in a similar principle of Michelson interferometer, except that collimated light beams travel through each arm only once. The flexibility of Mach-Zehnder interferometry to image over relatively large area makes it convenient for flow visualization in wind tunnels and plasma chambers (for instance, at the Alcator C-Mod Tokamak Located in building NW-21 on the MIT campus).

    a) As a practice problem, lets' consider a Mach-Zehnder interferometer with a gas cell of length 10 cm in one arm as shown. The input light wavelength is λ = 633 nm.
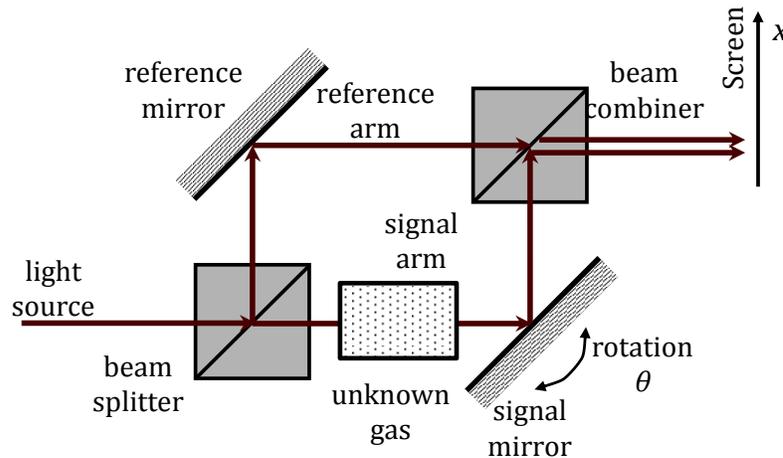
    The cell is initially evacuated, and we measure a bright spot on the screen at x=0 position. As the cell is pressurized, you observe that the spot at x=0 goes dark then becomes bright again with a total of 10 complete cycles. What is the refractive index of the gas at the final pressure?

    b) (**2.710 only**) Now let's assume the input light source replaced by a Na$^+$ lamp (bichromatic, corresponding to wavelength λ_1=589.592nm, λ_2=588.995nm). Since the splitting is small, $\Delta\lambda \equiv (\lambda_1 - \lambda_2) \ll \bar{\lambda} \equiv \frac{\lambda_1 + \lambda_2}{2}$, show that the intensity at x=0 varies periodically from bright to dark when the cell is pressurized with gas, starting again from vacuum, and express the accuracy of the measured refractive index as a function of $\bar{\lambda}^2/\Delta\lambda$.

2. (**Pedrotti 8-12**) White light is passed through a Fabry-Perot interferometer in the arrangement shown as follows, where the detector is a spectroscope. A series of bright bands appear. When mercury light is simultaneously admitted into the spectroscope slit, 150 of the bright bands are seen to fall between the violet and green lines of mercury at 435.8nm and 546.1nm, respectively. What is the thickness d of the Fabry-Perot cavity(etalon)?
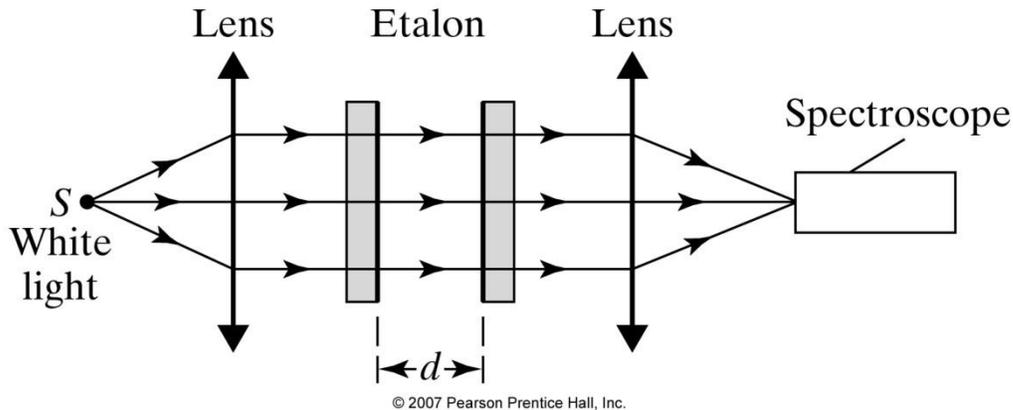
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3. Exercise on Fourier Transforms: Using known transform pairs and theorems, find the Fourier transforms of the following:

   a) $\text{rect}\left(\frac{x}{2W}\right)\text{rect}\left(\frac{y}{2W}\right)$

   b) $exp\left(-\frac{x^2+y^2}{2W^2}\right)$

   c) $\text{circ}\left(\frac{\sqrt{x^2+y^2}}{W}\right)$

   d) $\text{sinc}(x)\text{comb}(y) \otimes comb(\frac{x}{2})sinc\left(\frac{y}{4}\right)$

   e) $circ\left(\sqrt{(x-a)^2+y^2}\right) + circ\left(\sqrt{(x+a)^2+y^2}\right)$

4. Fast **Fourier Transform with MIT Seal!** Please download an MIT seal (similar to the attached example), convert it to grayscale by using one of R, G, B color value at each pixel, and crop its central portion $g(x, y)$ so that it have square shape (e.g., 400 × 400.) (Please read Voelz Chapter 2 and 3 regarding digital signal processing and FFT with MATLAB).

a) Plot your image next to the amplitude $|G(u, v)|$ *and phase* $arg(G(u, v))$ of its Fourier transform $G(u, v)$. (more details may be visible if instead you plot $log_{10}|G(u, v)|$.)

b) *Amplitude or Phase?* Let's assume one of the above two Fourier spectrum is immersed in the noise, so we can only recover amplitude or phase information, while the other is a constant(say $|G(u, v)|$ *becomes 1 everywhere*). Please plot the recovered image based only on $|G(u, v)|$ *or* $arg(G(u, v))$. Which one is closer to the original barcode?

c) To add a filter to this image, we define a 5 × 5 square region S around the origin of the ("shifted" in FFT case) Fourier transform domain:
$$T(u,v)=1 \text{ inside } S$$
$$T(u,v)=0 \text{ outside } S$$

Now plot the filtered image: $g1(x, y) = \iint G(u, v)T(u, v) \exp(-iux) \exp(-ivy) du dv$

d) Now invert the filter, so
$$H(u,v)=0 \text{ inside } S$$
$$H(u,v)=1 \text{ outside } S.$$

Plot the filtered image $g2(x, y) = \iint G(u, v)H(u, v) \exp(-iux) \exp(-ivy) du dv$.

If you use MATLAB to solve this problem, you will find the following functions useful: (i) **fft2** computes the 2D Fourier transform of an image and returns it with some quadrants swapped, (ii) **fftshift** rearranges the quadrants of the Fourier transform in their proper order, (iii) **ifft2** computes the inverse 2D Fourier transform (iv) **imagesc**; **colormap** gray displays a real grayscale image, (v) **print -dps** [filename] prints a figure into a postscript file which you can then print at any Athena printer using the **lpr** command.

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