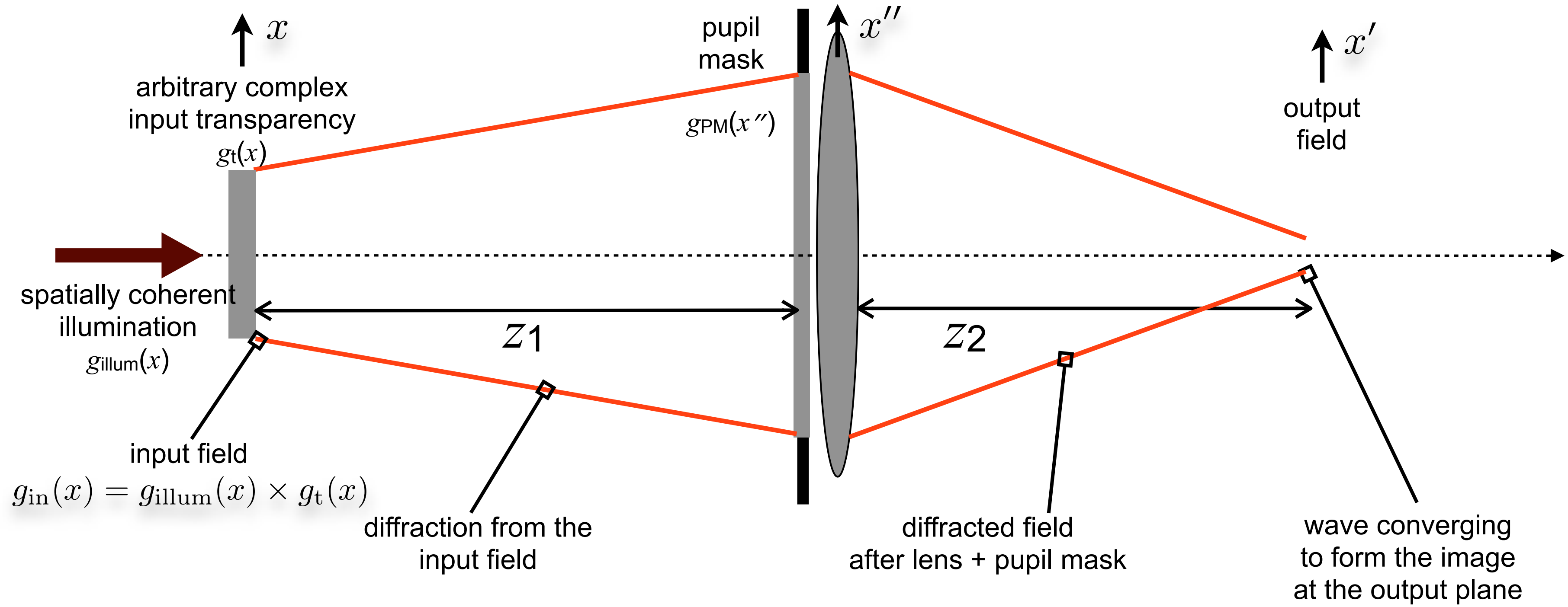
Today

- Spatially coherent and incoherent imaging with a single lens
  - re-derivation of the single-lens imaging condition
  - ATF/OTF/PSF and the Numerical Aperture
  - resolution in optical systems
  - pupil engineering revisited

next week

- Two more applications of the MTF
  - defocus
  - diffractive optics and holography
- Multi-pass interferometers: Fabry-Perot
  - optical resonators and Lasers
- Beyond scalar optics: polarization and resolution

Imaging with a single lens: imaging condition



The field at the output plane of this imaging system is (derivation in the supplement to this lecture)

$$g_{out}(x',y') = -\frac{1}{\lambda^2 z_1 z_2}\exp\left\{i2\pi\frac{z_1+z_2}{\lambda}+i\pi\frac{x'^2+y'^2}{\lambda z_2}\right\}\iint g_{in}(x,y)\exp\left\{i\pi\frac{x^2+y^2}{\lambda z_1}\right\}$$
$$\times\left[\iint g_{PM}(x'',y'')\exp\left\{i\pi\frac{x''^2+y''^2}{\lambda}\left(\frac{1}{z_1}+\frac{1}{z_2}-\frac{1}{f}\right)-i2\pi\frac{x''}{\lambda}\left(\frac{x}{z_1}+\frac{x'}{z_2}\right)-i2\pi\frac{y''}{\lambda}\left(\frac{y}{z_1}+\frac{y'}{z_2}\right)\right\}\mathrm{d}x''\mathrm{d}y''\right]\mathrm{d}x\mathrm{d}y$$

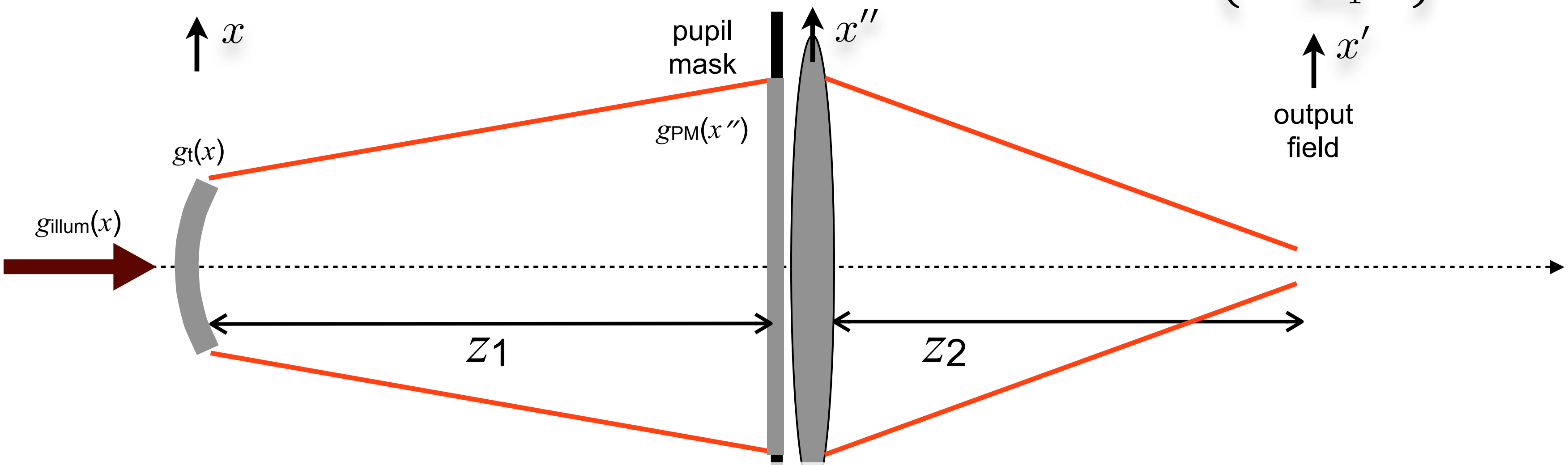
We also need to eliminate this quadratic term

To eliminate the quadratic term we satisfy the Lens Law of geometrical optics

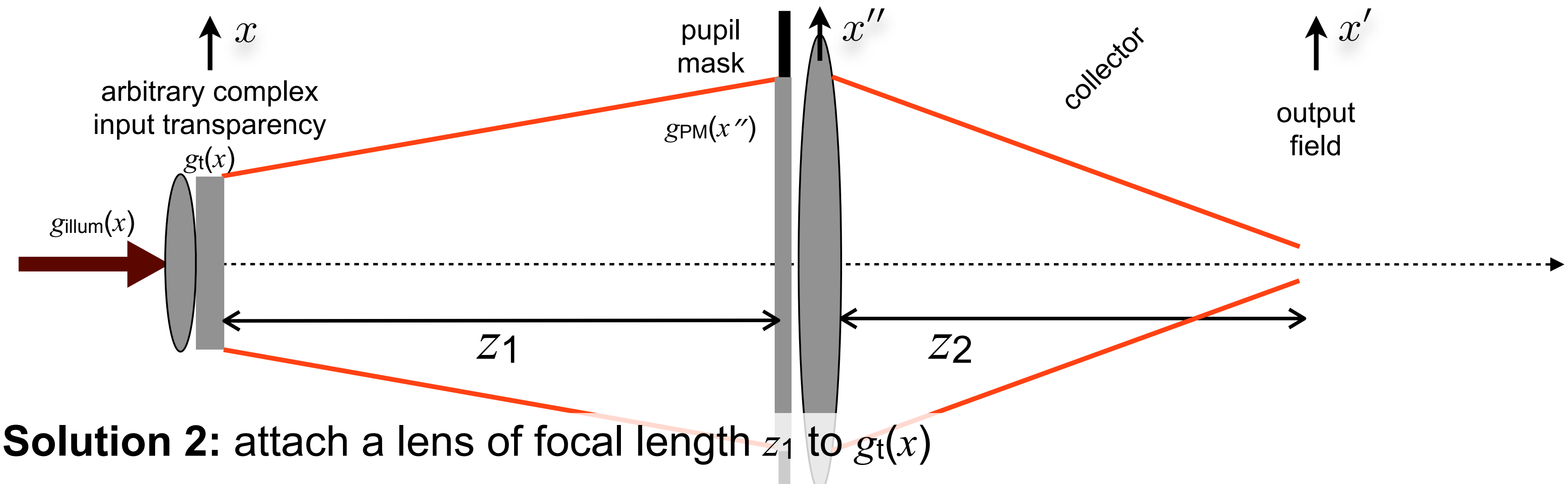
$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$$

After eliminating the quadratic, the remaining integral is the Fourier transform of the pupil mask $g_{PM}(x'', y'')$.

Eliminating the quadratic term $\exp\left\{i\frac{x^2+y^2}{\lambda z_1}\right\}$



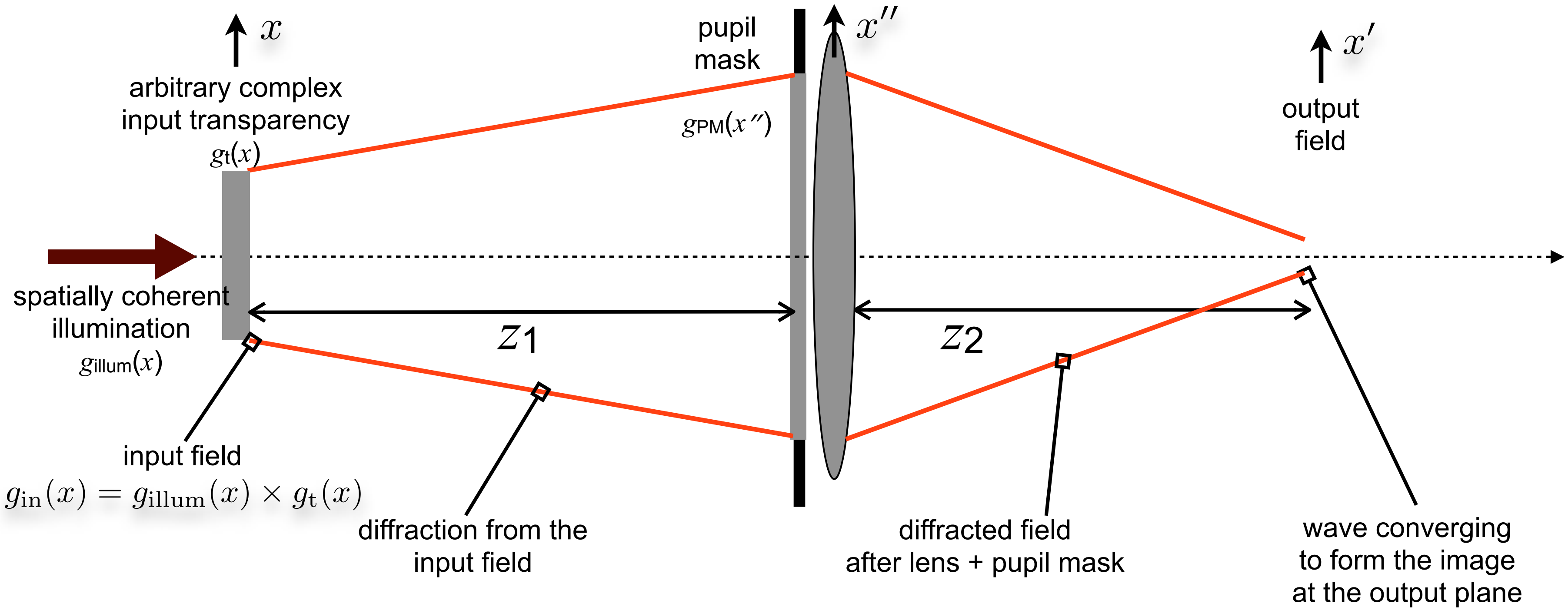
Solution 1: shape $g_t(x)$ as a sphere of radius z_1



Solution 2: attach a lens of focal length z_1 to $g_t(x)$

Solution 3: limit $g_t(x)$ to ¼ the lateral size of $g_{PM}(x'')$ [see Goodman 5.3.2 and Ref. 303]

Imaging with a single lens: PSF and ATF



Assuming one of the three conditions is satisfied and the last remaining quadratic term can be eliminated, the output field is

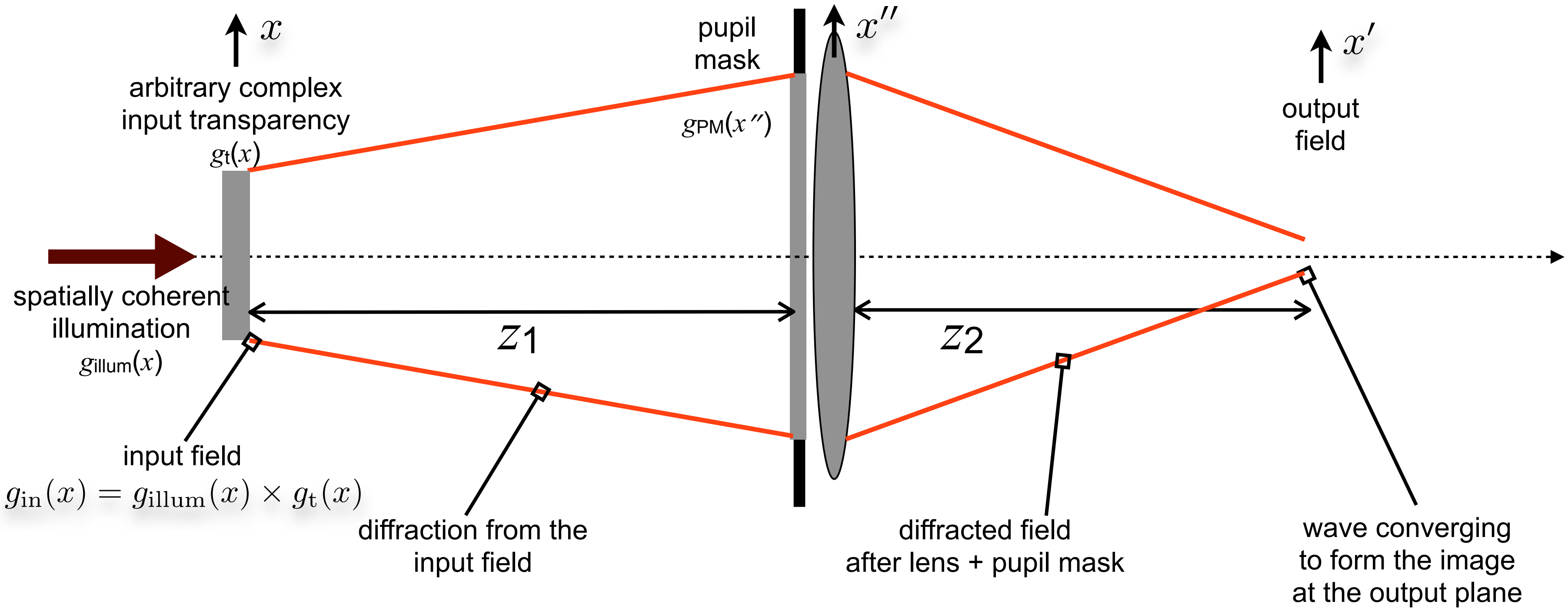
$$g_{\mathrm{out}}(x',y') = -\exp\left\{i2\pi\frac{z_1+z_2}{\lambda} + i\pi\frac{x'^2+y'^2}{\lambda z_2}\right\}\left(\frac{z_1}{z_2}\right)\iint g_{\mathrm{in}}(x,y)\,h\left(x+\frac{z_1}{z_2}x',\, y+\frac{z_1}{z_2}y'\right)\mathrm{d}x\mathrm{d}y$$

where $h(x,y) \equiv (\lambda z_1)^2 \iint g_{\mathrm{PM}}(\lambda z_1 u, \lambda z_1 v)\exp\left\{-i2\pi(ux+vy)\right\}\mathrm{d}x\mathrm{d}y$ is the PSF,

i.e. the Fourier transform of the pupil mask scaled so that $(x'', y'') = (\lambda z_1 u, \lambda z_1 v)$. As in the 4F system,

the scaled complex transmissivity of the pupil mask is the ATF $H(u,v) \equiv (\lambda z_1)^2\, g_{\mathrm{PM}}(\lambda z_1 u, \lambda z_1 v)$

Imaging with a single lens: lateral magnification



If the pupil mask $g_{PM}(x'', y'')$ is infinitely large and clear, its Fourier transform is approximated as a δ-function. Therefore, the optical field at the output plane is

$$g_{out}(x', y') \propto \exp\left\{ i\pi \frac{x'^2 + y'^2}{\lambda z_2} \left(1 + \frac{z_1}{z_2}\right) \right\} g_{in}\left(-\frac{z_1}{z_2} x', -\frac{z_1}{z_2} y' \right)$$

phase factor, does not affect the image intensity

replica of the input field within the factor of lateral magnification

$$M_T = -\frac{z_2}{z_1}$$

So by ignoring diffraction due to the finite lateral size and, possibly, phase-delay elements inside the clear aperture of the pupil mask $g_{PM}(x'', y'')$, we have essentially found that the imaging condition and lateral magnification relationships from geometrical optics remain valid in wave optics as well for the *intensity* of the optical field.

Block diagrams for coherent and incoherent linear shift invariant imaging systems

spatially coherent illumination

$g_{\mathrm{illum}}(x,y)$ → thin transparency $g_{\mathrm{t}}(x,y)$ →

input field:
$g_{\mathrm{in}}(x,y) =$
$= g_{\mathrm{illum}}(x,y) \times g_{\mathrm{t}}(x,y)$

cPSF ⇒ **convolution**

output field:
$g_{\mathrm{out}}(x',y') =$
$= g_{\mathrm{in}}(x,y) \circledast h(x,y)$

⇕ Fourier transform

$G_{\mathrm{in}}(u,v)$

ATF ⇒ multiplication

$G_{\mathrm{out}}(u,v) =$
$= G_{\mathrm{in}}(u,v) \times H(u,v)$

4F imaging system:
$H(u,v) \equiv (\lambda f_1)^2 \, g_{\mathrm{PM}}(\lambda f_1 u, \lambda f_1 v)$

Single lens imaging system:
$H(u,v) \equiv (\lambda z_1)^2 \, g_{\mathrm{PM}}(\lambda z_1 u, \lambda z_1 v)$

---

spatially *in*coherent illumination

$I_{\mathrm{illum}}(x,y)$ → thin transparency $g_{\mathrm{t}}(x,y)$ →

input intensity:
$g_{\mathrm{I}}^{\mathrm{in}}(x,y) =$
$= I_{\mathrm{illum}}(x,y) \times |g_{\mathrm{t}}(x,y)|^2$

iPSF ⇒ **convolution**

output intensity:
$g_{\mathrm{I}}^{\mathrm{out}}(x',y') =$
$= g_{\mathrm{I}}^{\mathrm{in}}(x,y) \circledast h_{\mathrm{I}}(x,y)$

⇕ Fourier transform

$G_{\mathrm{I}}^{\mathrm{in}}(u,v)$

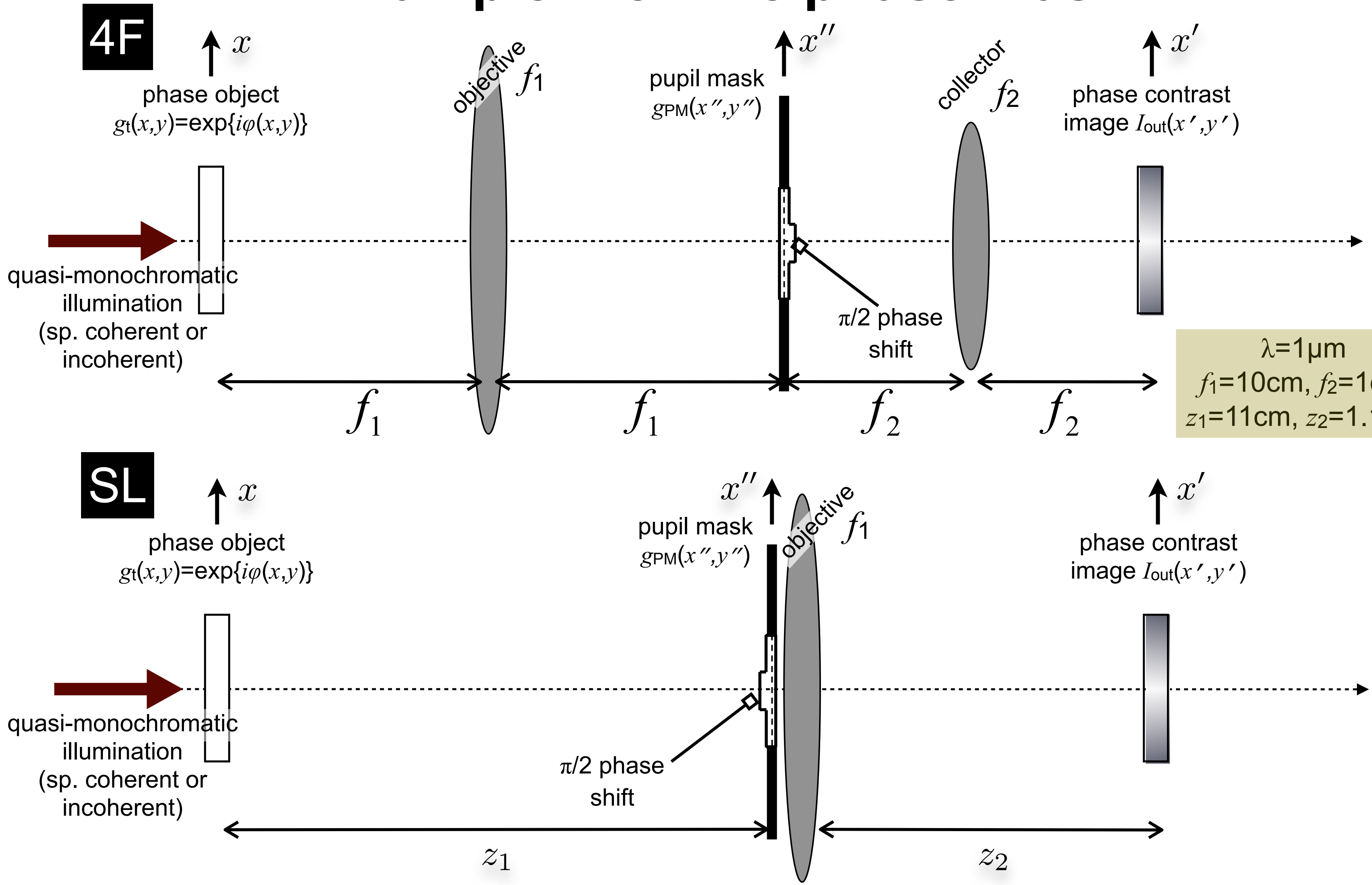
OTF ⇒ multiplication

$G_{\mathrm{I}}^{\mathrm{out}}(u,v) =$
$= G_{\mathrm{I}}^{\mathrm{in}}(u,v) \times \mathcal{H}(u,v)$

4F imaging system:
$\mathcal{H}(u,v) \propto (g_{\mathrm{PM}} \star g_{\mathrm{PM}}) \big|_{(\lambda f_1 u, \lambda f_1 v)}$

Single lens imaging system:
$\mathcal{H}(u,v) \propto (g_{\mathrm{PM}} \star g_{\mathrm{PM}}) \big|_{(\lambda z_1 u, \lambda z_1 v)}$

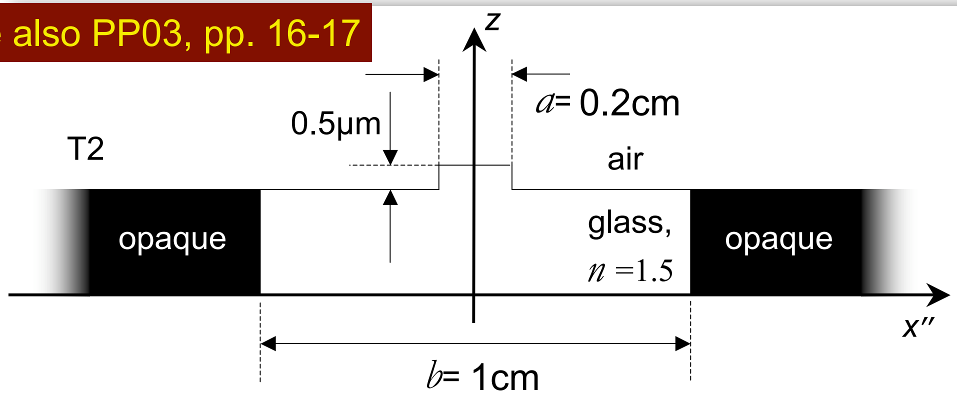
Example: Zernike phase mask



$\lambda=1\mu m$
$f_1=10cm$, $f_2=1cm$
$z_1=11cm$, $z_2=1.1cm$

ATF and OTF of the Zernike phase mask

See also PP03, pp. 16-17



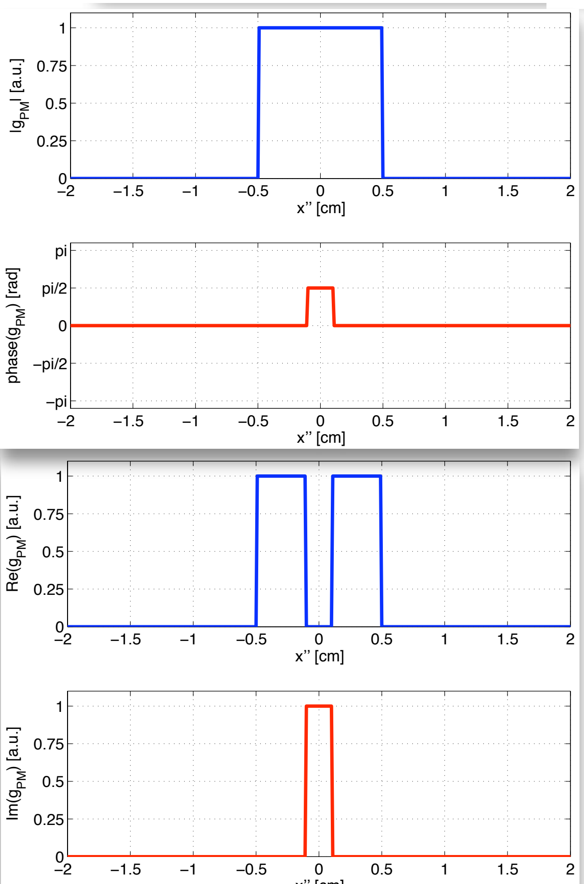
$$g_{\mathrm{PM}}\left(x'', y''\right) = \mathrm{rect}\left(\frac{x''}{1\mathrm{cm}}\right) + \left(\mathrm{e}^{i\pi/2} - 1\right)\mathrm{rect}\left(\frac{x''}{0.2\mathrm{cm}}\right)$$

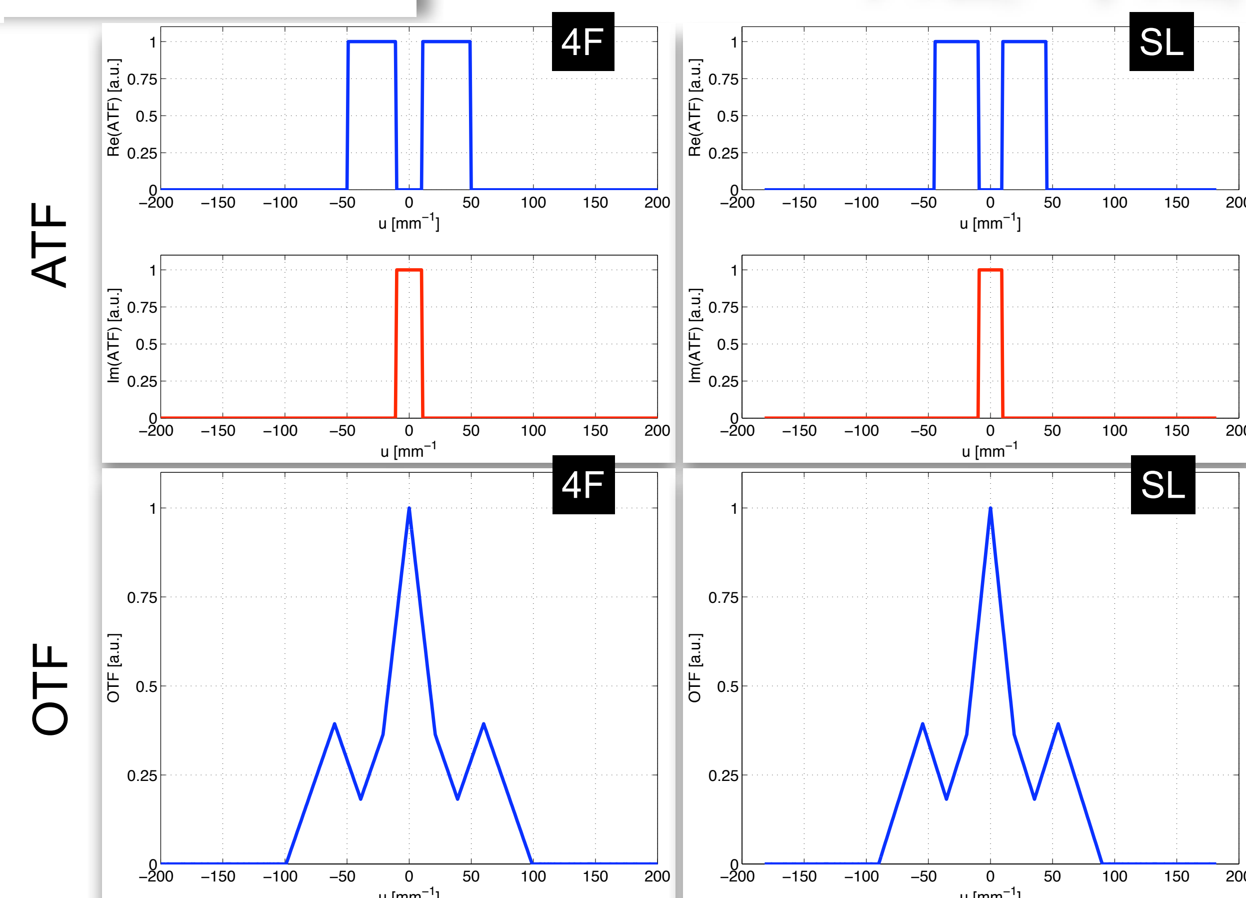
4F system:

$$x'' = u\lambda f_1, \qquad y'' = v\lambda f_2$$

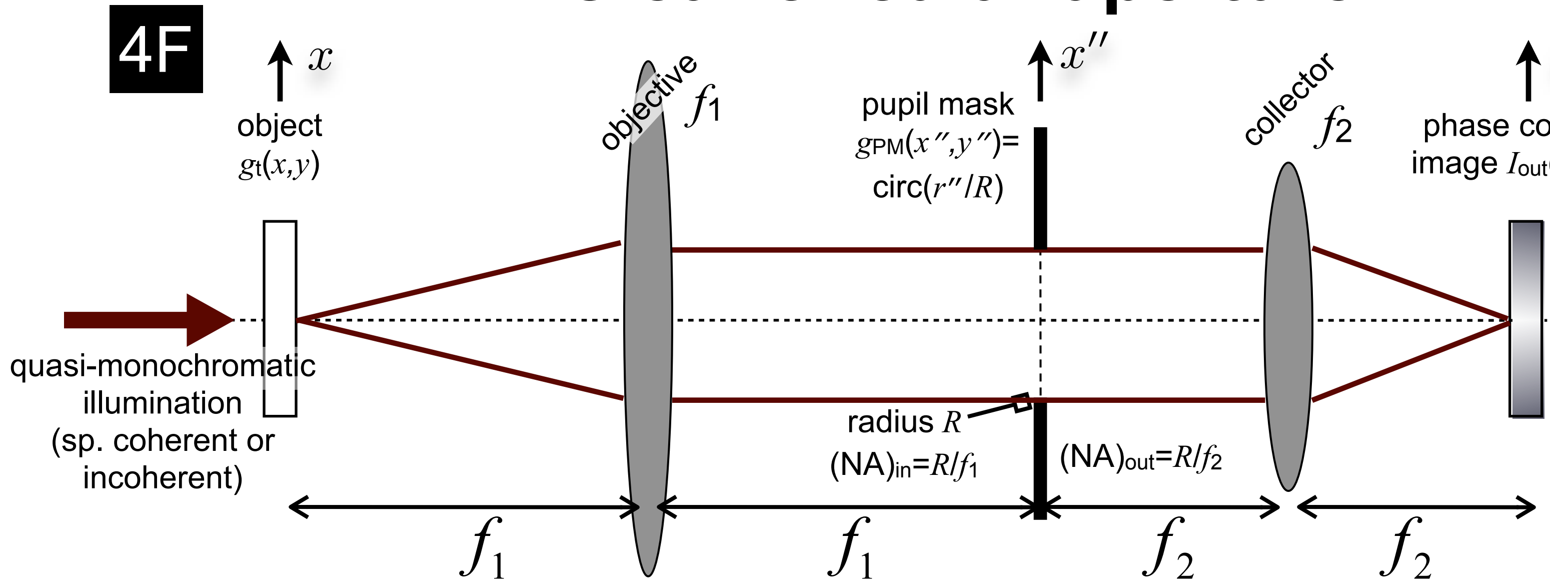
Single Lens (SL) system:

$$x'' = u\lambda z_1, \qquad y'' = v\lambda z_2$$

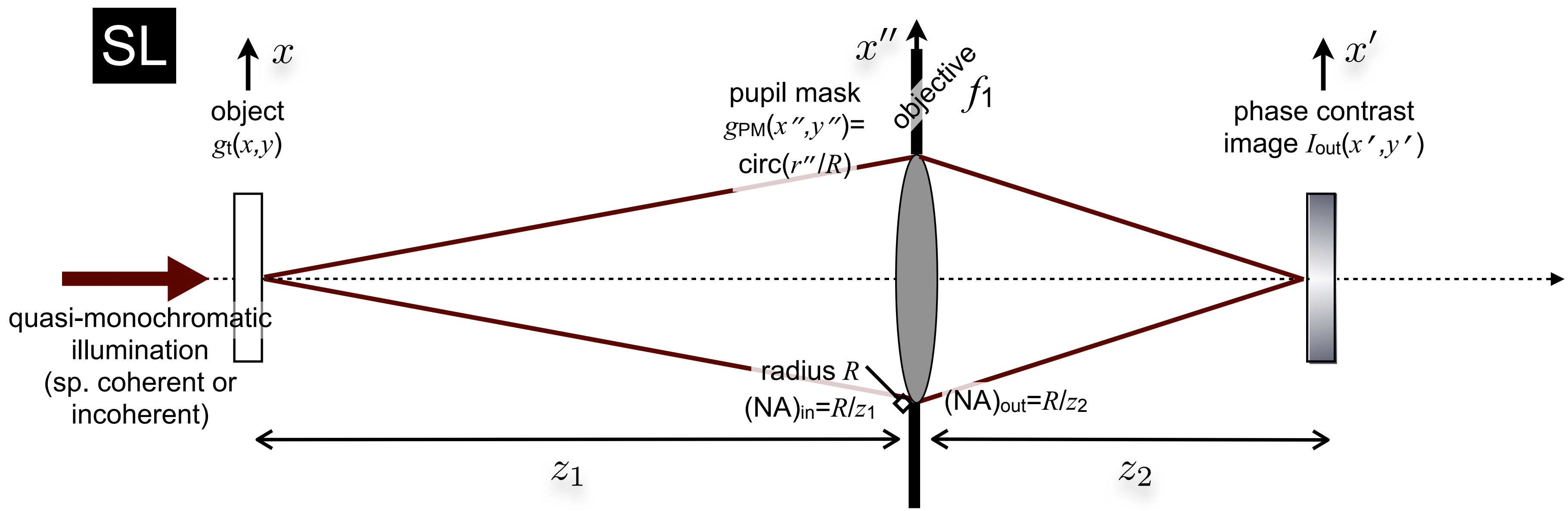




Clear circular aperture



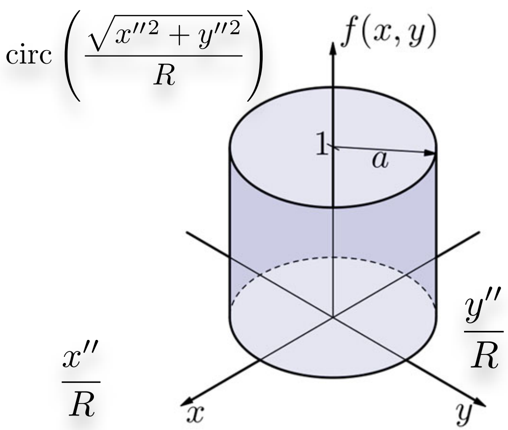
λ=1μm
f_1=10cm, f_2=1cm
z_1=cm, z_2=cm



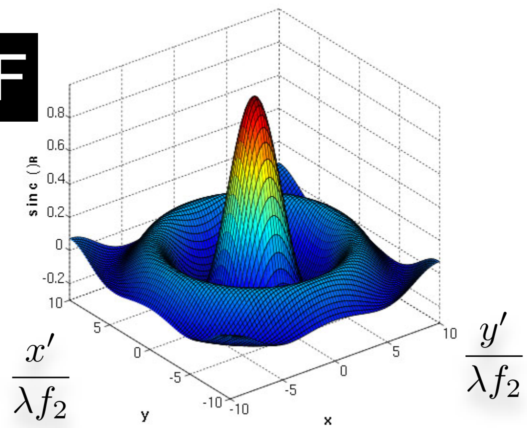
ATF, cPSF of clear circular aperture

$$\mathcal{F}\left\{\operatorname{circ}\left(\frac{r''}{R}\right)\right\} = R\frac{\mathrm{J}_1(2\pi R\rho)}{R\rho} \equiv R\,\mathrm{jinc}(R\rho)$$

$$r'' = \sqrt{x''^2 + y''^2}$$
$$\rho = \sqrt{u^2 + v^2}$$

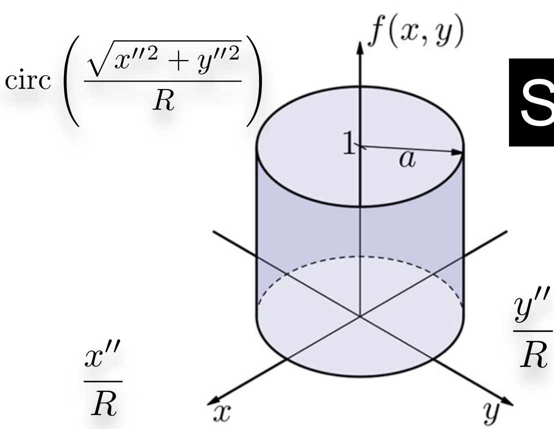


4F

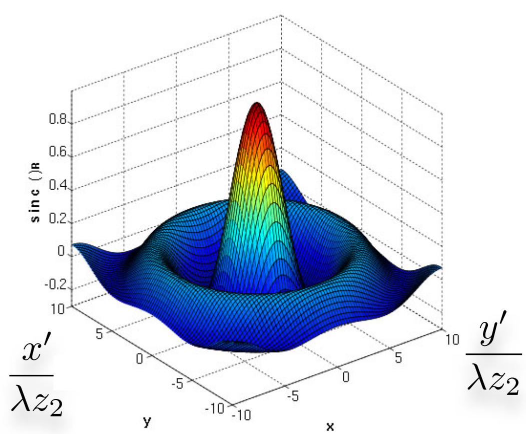


$$u = x''/(\lambda f_2)$$
$$v = y''/(\lambda f_2)$$

$$h(x', y') = R\,\mathrm{jinc}\left(\frac{R\sqrt{x'^2 + y'^2}}{\lambda f_2}\right)$$



SL



$$u = x''/(\lambda z_2)$$
$$v = y''/(\lambda z_2)$$

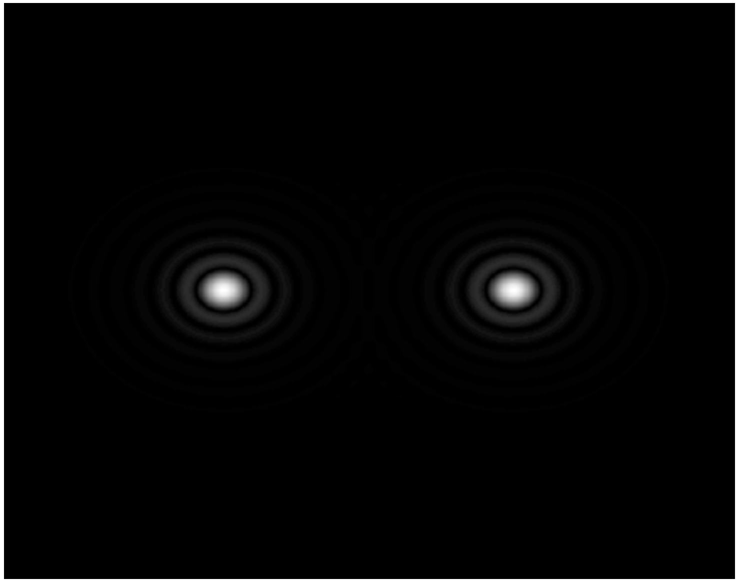
$$h(x', y') = R\,\mathrm{jinc}\left(\frac{R\sqrt{x'^2 + y'^2}}{\lambda z_2}\right)$$

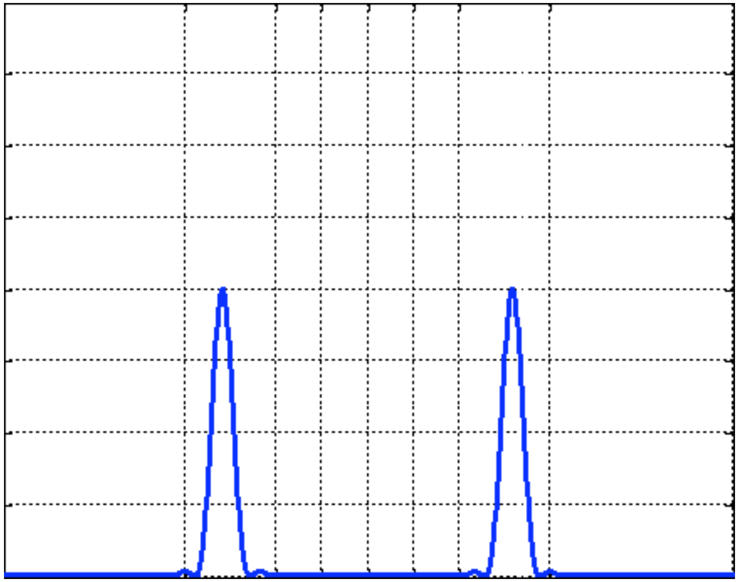
Resolution

[from the New Merriam-Webster Dictionary, 1989 ed.]:

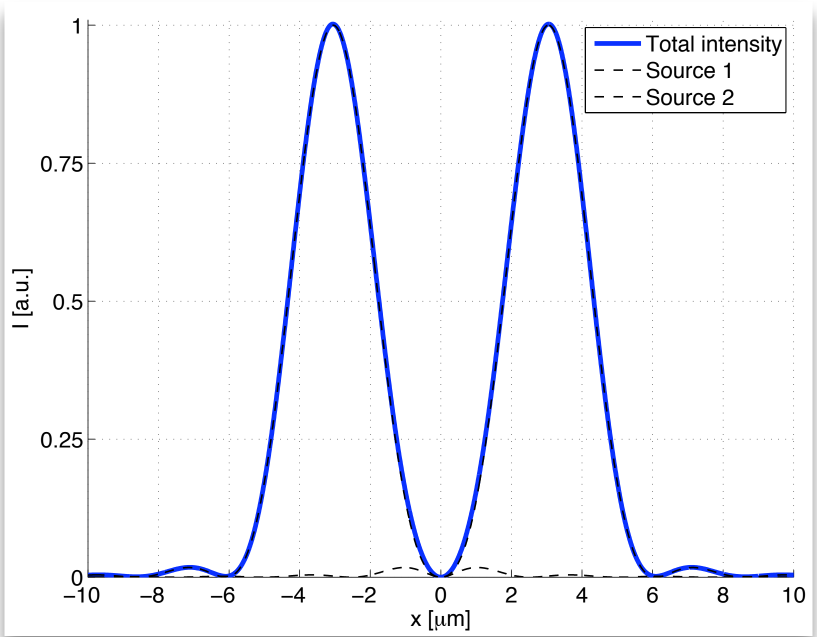
resolve *v* : **1** to break up into constituent parts: ANALYZE; **2** to find an answer to : SOLVE; **3** DETERMINE, DECIDE; **4** to make or pass a formal resolution

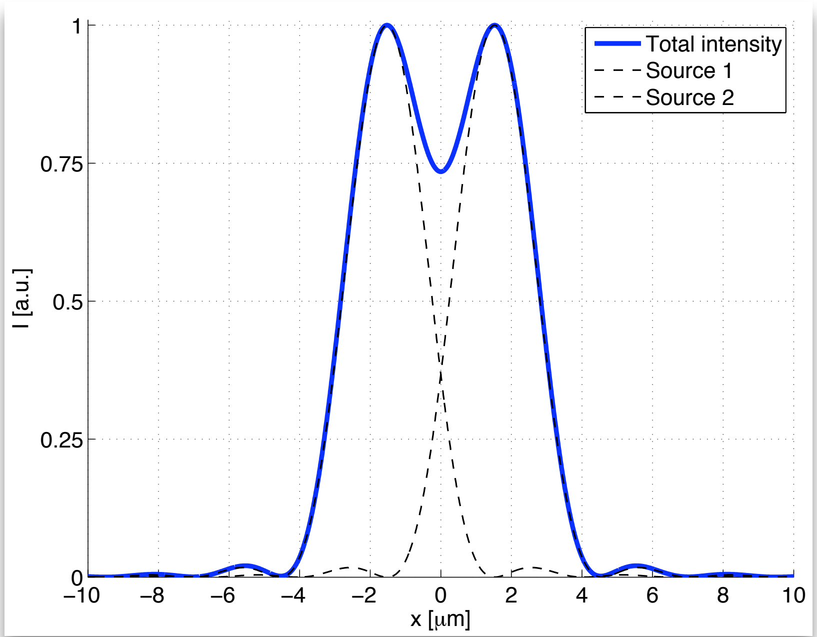
resolution *n* : **1** the act or process of resolving **2** the action of solving, *also* : SOLUTION; **3** the quality of being resolute: FIRMNESS, DETERMINATION; **4** a formal statement expressing the opinion, will or, intent of a body of persons





Rayleigh resolution limit





Two point sources are well resolved if they are spaced such that:

(i) the PSF *diameter*
equals the point source spacing

$$\Delta r = 1.22\frac{\lambda}{(\mathrm{NA})_{\mathrm{in}}}$$

$$\Delta r' = 1.22\frac{\lambda}{(\mathrm{NA})_{\mathrm{out}}}$$

(i) the PSF *radius*
equals the point source spacing

$$\Delta r = 0.61\frac{\lambda}{(\mathrm{NA})_{\mathrm{in}}}$$

$$\Delta r' = 0.61\frac{\lambda}{(\mathrm{NA})_{\mathrm{out}}}$$

2.71 / 2.710 Optics
Spring 2009