## Homework V solution

## Problem 1.

1. The reaction rate equations for the three reactions are as follow:
$k_{b, 1}=k_{f, 1} / K_{C 1}$
$k_{b, 2}=k_{f, 2} / K_{C 2}$
$R_{r, 1}=k_{f, 1}[\mathrm{CO}]\left[\mathrm{H}_{2}\right]^{2}-k_{b, 1}\left[\mathrm{CH}_{3} \mathrm{OH}\right]$
$R_{r, 2}=k_{f, 2}[\mathrm{CO}]\left[\mathrm{H}_{2} \mathrm{O}\right]-k_{b, 2}\left[\mathrm{CO}_{2}\right]\left[\mathrm{H}_{2}\right]$
$R_{r, 3}=k_{f, 3}\left[\mathrm{CH}_{3} \mathrm{OH}\right]$
Note that you should consider backward reactions in the first two reactions.
2. The rates of formation of each species are as follow:

$$
\begin{aligned}
& \frac{d[\mathrm{CO}]}{d t}=-R_{r, 1}-R_{r, 2} \\
& \frac{d\left[\mathrm{H}_{2}\right]}{d t}=-2 R_{r, 1}+R_{r, 2}+R_{r, 3} \\
& \frac{d[\mathrm{CH} \mathrm{OH}]}{d t}=R_{r, 1}-R_{r, 3} \\
& \frac{d\left[\mathrm{H}_{2} \mathrm{O}\right]}{d t}=-R_{r, 2} \\
& \frac{d\left[\mathrm{CO}_{2}\right]}{d t}=R_{r, 2} \\
& \frac{d\left[\mathrm{CH}_{2} \mathrm{O}\right]}{d t}=R_{r, 3}
\end{aligned}
$$

3. 



4. Increasing the pressure raises the mole fraction of CH 3 OH when $\mathrm{T} \leq 340$, but does not have a significant effect after $\mathrm{T} \geq 360$. It is due to the following reasons:

1) In the first reaction, which generates $\mathrm{CH}_{3} \mathrm{OH}$, the forward reaction favors higher pressure since the number of moles decreases during the forward reaction. Thus, by increasing the pressure of the reactor, the concentration of $\mathrm{CH}_{3} \mathrm{OH}$ increases more rapidly.
2) After $T \geq 360$, even at low pressure, the first reaction is sufficiently fast. So increasing the pressure does not help. Also, note that the concentration of $\mathrm{CH}_{3} \mathrm{OH}$ drops back to almost zero with time due to the third reaction (dissociation of CH 3 OH ) indicating that the third reaction becomes active after $\mathrm{T} \geq 360$.
5. The maximum mole fraction of $\mathrm{CH}_{3} \mathrm{OH}$ one can obtain is $\sim 33 \%$ at $\mathrm{T}=340 \mathrm{~K}$ and $\mathrm{P}=10$ 100 atm with a residence time less than 1 msec .
6. The mole concentration changes at $\mathrm{T}=340 \mathrm{~K}, \mathrm{P}=10 \mathrm{~atm}$ and residence time $=1 \mathrm{msec}$ are as follow:

|  | CO | $\mathrm{H}_{2}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | H 2 O | $\mathrm{CO}_{2}$ | $\mathrm{CH}_{2} \mathrm{O}$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Before the <br> reactor <br> $\left(\mathrm{mol} / \mathrm{m}^{3}\right)$ | 71.5 | 167 | 0 | 47.6 | 71.5 | 0 | 357.3 |
| After the <br> reactor <br> $\left(\mathrm{mol} / \mathrm{m}^{3}\right)$ | 0.0 | 25.8 | 70.6 | 47.1 | 72.0 | 0.3 | 215.8 |

From the first law of the thermodynamics one can get Q as follow:
$Q=H_{2}-H_{1}=\sum_{P} n_{i} \hat{h}_{i}-\sum_{R} n_{i} \hat{h}_{i}=-6.5 \mathrm{MJ}$
Since this Q is with 357.3 mole of inlet mixture, one can get $\dot{Q}$ as follow:
$\dot{Q}=-6.5 \mathrm{MJ} / 357.3$ moles $\cdot 300 \mathrm{moles} / \mathrm{s}=-5.4 \mathrm{Mwatt}$

## Problem 2.

1. $j_{o}=i_{o} / A=k^{o}[O]^{*} F=10^{-6} \mathrm{~cm} / \mathrm{s} \cdot 10^{-3} \mathrm{~mol} / 1000 \mathrm{~cm}^{3} \cdot 96485 \mathrm{Coulomb} / \mathrm{mol}=9.65 \cdot 10^{-2} \mu \mathrm{~A} / \mathrm{cm}^{2}$

2 and 3. Using the Butler-Volmer equation, you can get the following figures



