## MIT Department of Mechanical Engineering 2.25 Advanced Fluid Mechanics

## Vector Calculus Review Problems

- (1) Show that the condition for the vectors $\underline{a}, \underline{b}$, and $\underline{c}$ to be coplanar is:
$\varepsilon_{i j k} a_{i} b_{j} c_{k}=0$
- (2) Prove the following relationships:

$$
\delta_{i j} \delta_{i j}=3 \quad \varepsilon_{p q i} \varepsilon_{p q j}=2 \delta_{i j}
$$

- (3) Use Stokes theorem to prove that $\nabla \times(\nabla \phi)=0$ for any single-valued twice-differentiable scalar $(\phi)$ regardless of the coordinate system.
- (4) Problem 3.12 from Panton's Fourth Edition: Write the following formulas in Gibbs's notation using the symbol $\nabla$. Convert the expressions to Cartesian notation and prove that the equations are correct.

$$
\begin{gathered}
\operatorname{div}(\phi \underline{v})=\phi \operatorname{div} \underline{v}+\underline{v} \cdot \operatorname{grad} \phi \\
\operatorname{div}(\underline{u} \times \underline{v})=\underline{v} \cdot \operatorname{curl} \underline{u}-\underline{u} \cdot \operatorname{curl} \underline{v} \\
\operatorname{curl}(\underline{u} \times \underline{v})=\underline{v} \cdot \operatorname{grad} \underline{u}-\underline{u} \cdot \operatorname{grad} \underline{v}+\underline{u} \operatorname{div} \underline{v}-\underline{v} \operatorname{div} \underline{u}
\end{gathered}
$$

MIT OpenCourseWare
http://ocw.mit.edu

### 2.25 Advanced Fluid Mechanics

Fall 2013

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

