

## Homework 5

2.18/2.180

**Due Wednesday March 18 at beginning of class**

Problems with a star (\*) are for graduate students only.

**Problem 1:** (5 points) Exercise 3.3

**Problem 2:** (5 points) Consider the two abstract “motifs” of incoherent feedforward loops given in Section 3.2. Propose one implementation for each of the two motifs, using the core processes seen in Chapter 2 to derive the corresponding ODE models, and demonstrate that the implementations work. That is, show that the output  $y$  asymptotically reaches a constant value  $y_0$  that is independent of the disturbance input  $u$ . Make sure you clarify the assumptions that are important for the implementation to work.

**Problem 3(\*):** Consider the following model of a *nonlinear harmonic oscillator* (Fig 3.15a):

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1(1 - x_1^2 - x_2^2) \\ \dot{x}_2 &= -x_1 + x_2(1 - x_1^2 - x_2^2)\end{aligned}$$

Answer the following questions:

- (i) demonstrate that the system admits a limit cycle;
  - (ii) demonstrate that this limit cycle is asymptotically stable
- (Hint: using radial coordinates may be useful) .

Consider now the following modification to the above model:

$$\begin{aligned}\dot{x}_1 &= x_2^3 \\ \dot{x}_2 &= -x_1^3\end{aligned}$$

- (iii) Demonstrate that this system also admits a limit cycle
- (iv) Determine whether the limit cycle is stable, unstable, or asymptotically stable.

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