BLOCK DIAGRAMS, BOND GRAPHS AND CAUSALITY

The main purpose of modeling is to develop insight.

“Drawing a picture” of a model promotes insight.

Why not stick with the familiar block diagrams?

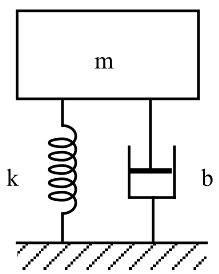
Block diagrams provide a picture of equations;

—they portray *operators* acting on *signals*.

Bond graphs are related to model equations,

—but there may be many different choices of equations to represent a given model;

—there may be many different block diagrams corresponding to one bond graph.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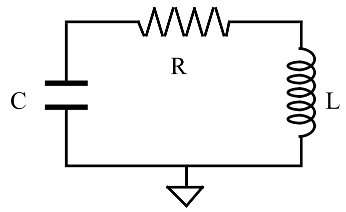
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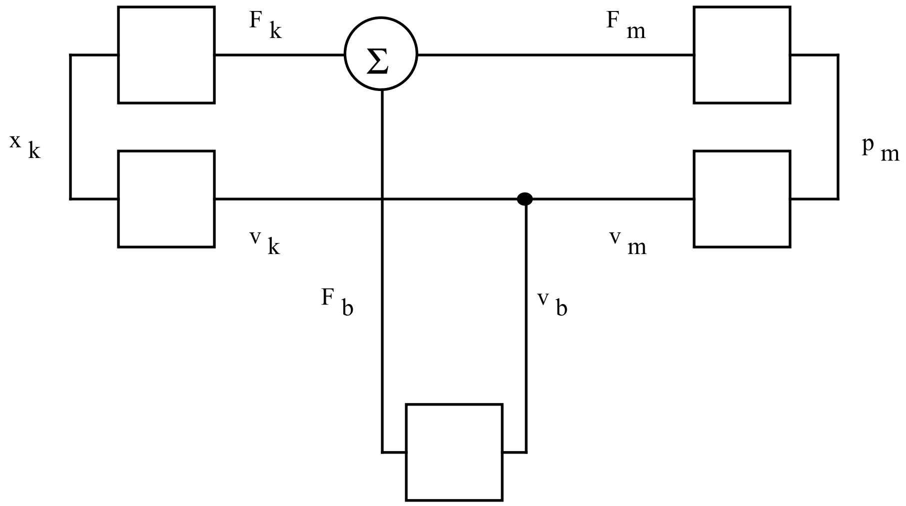
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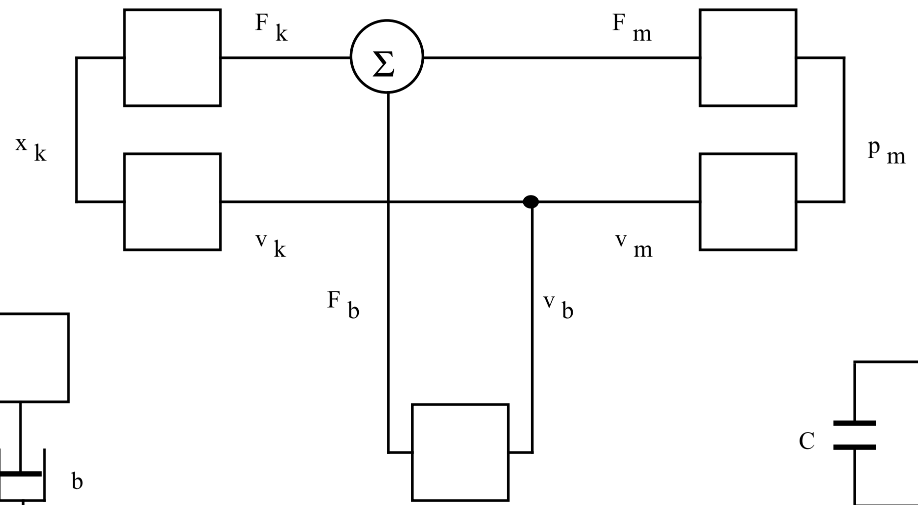
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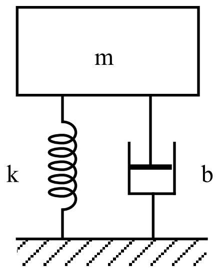
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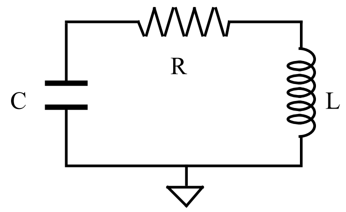
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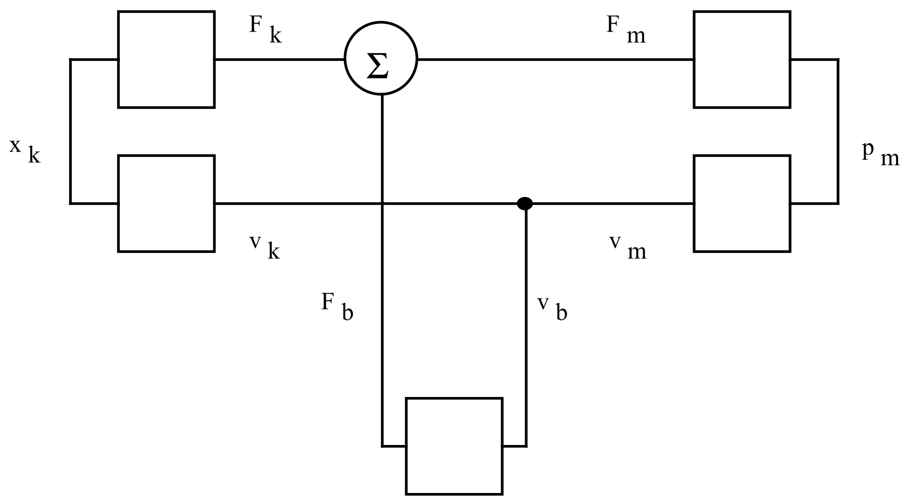
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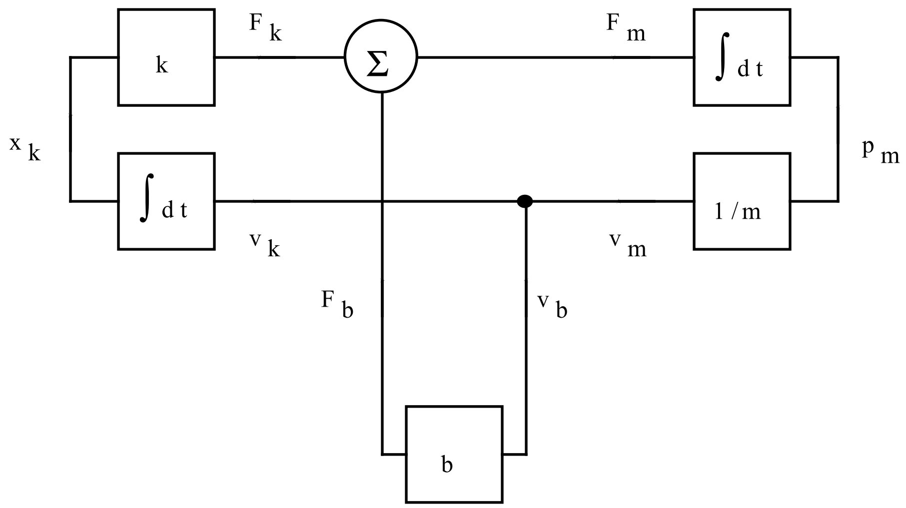
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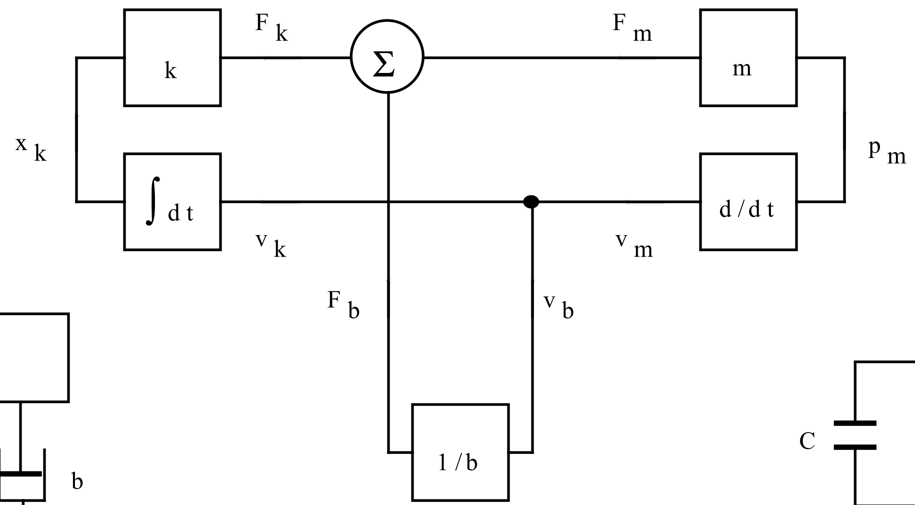
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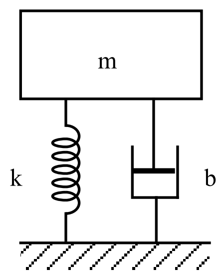
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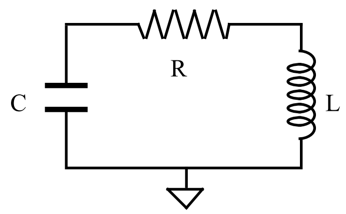
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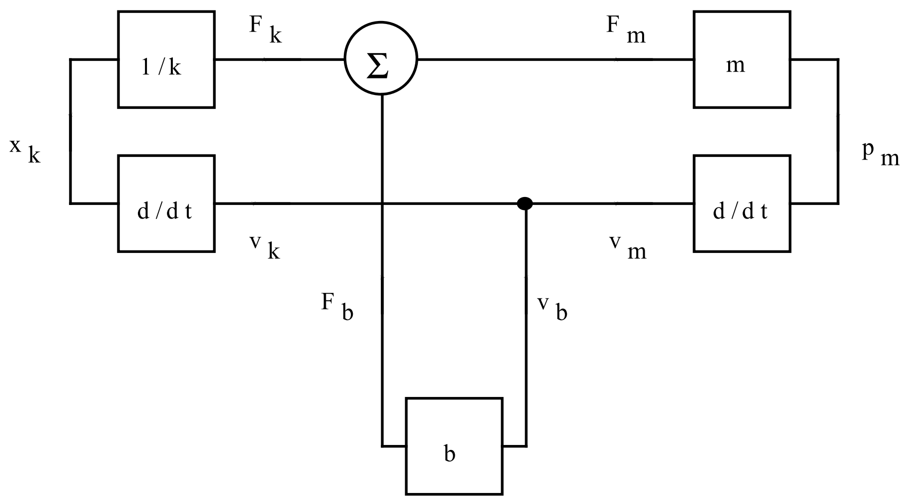
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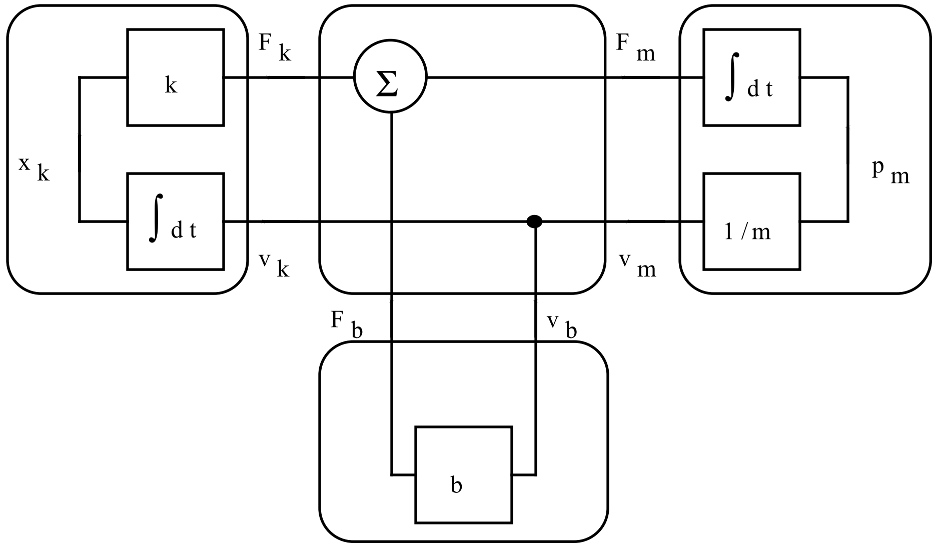
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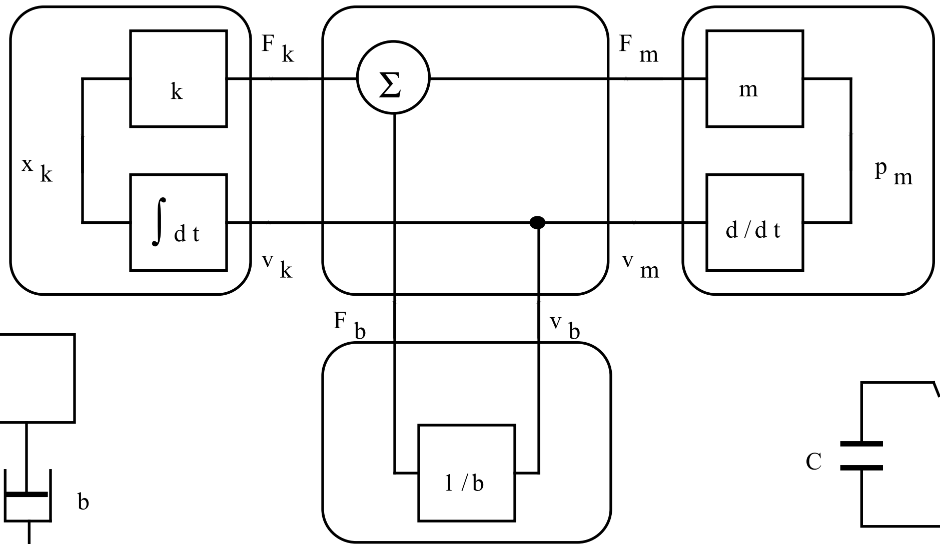
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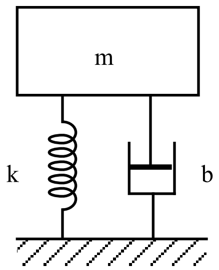
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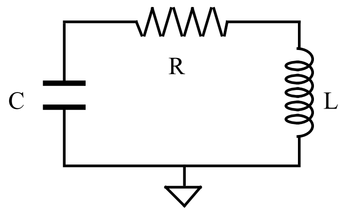
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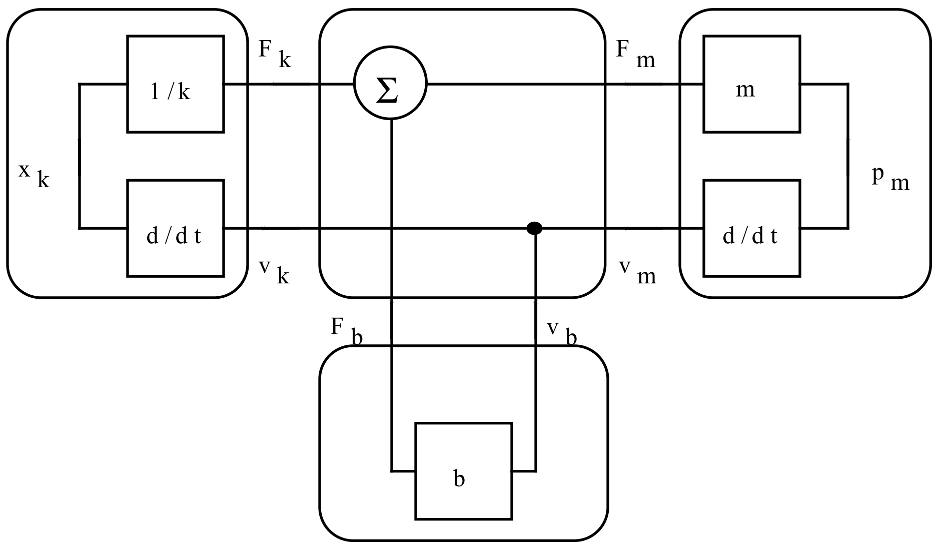
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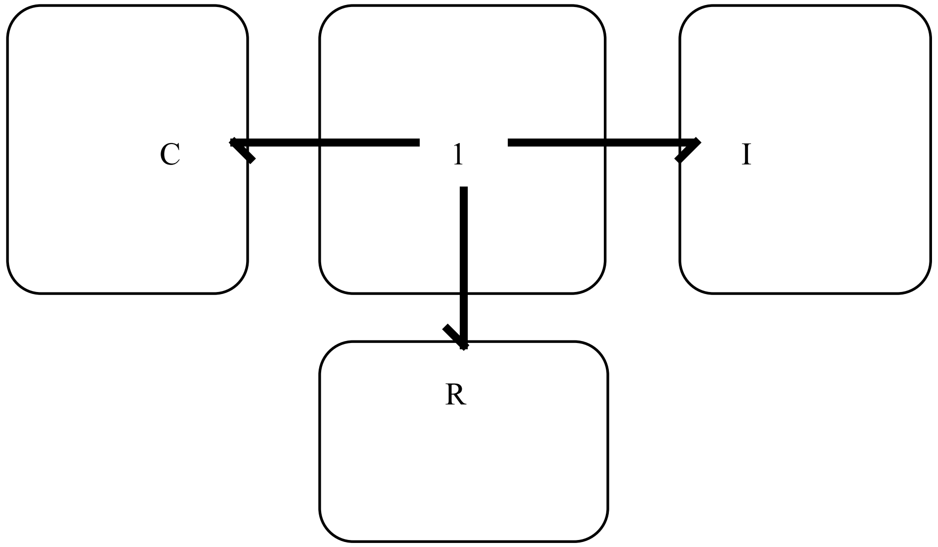
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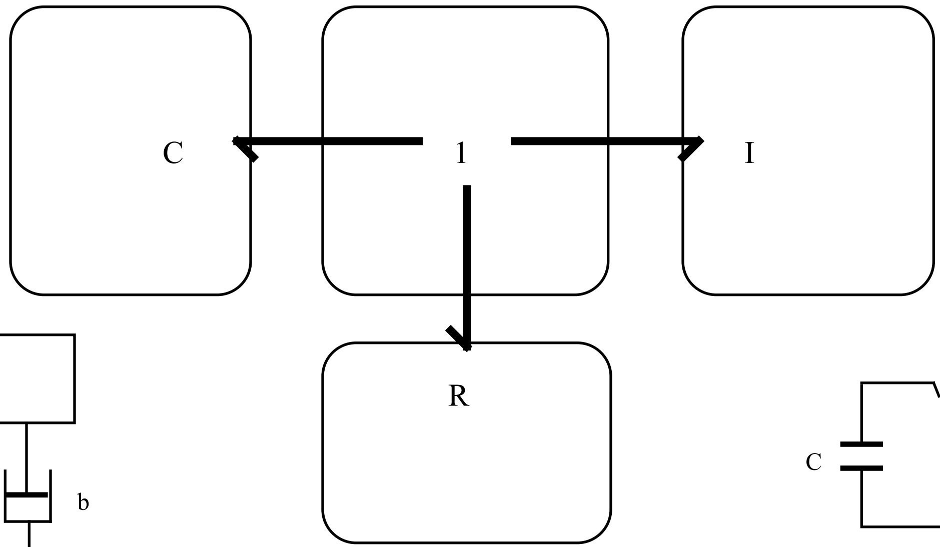
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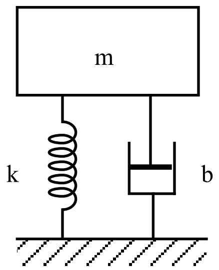
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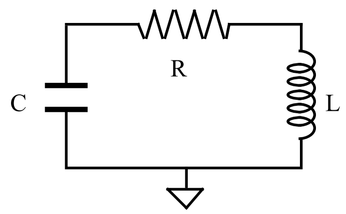
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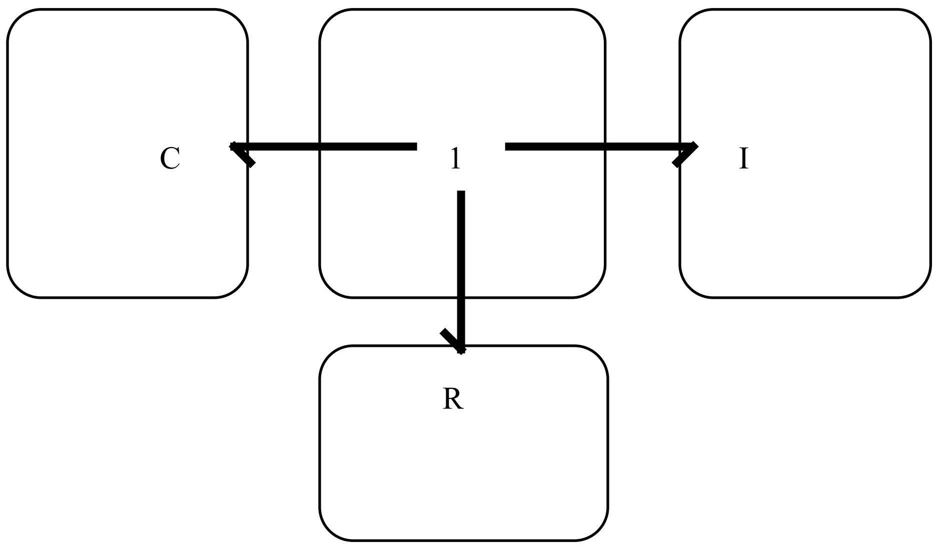
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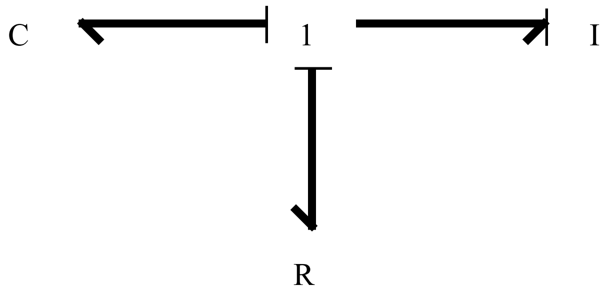
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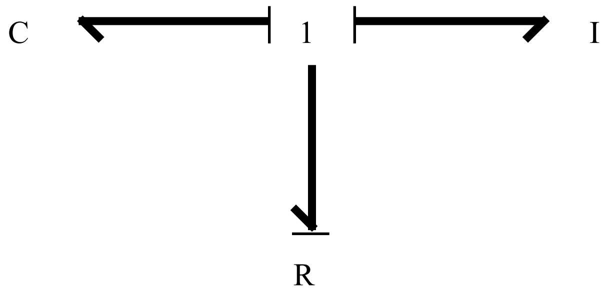
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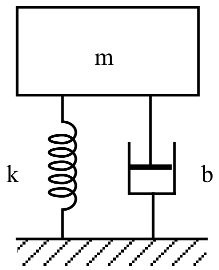
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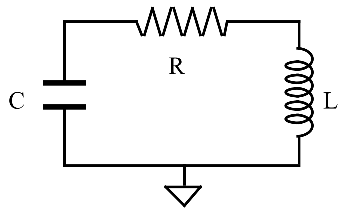
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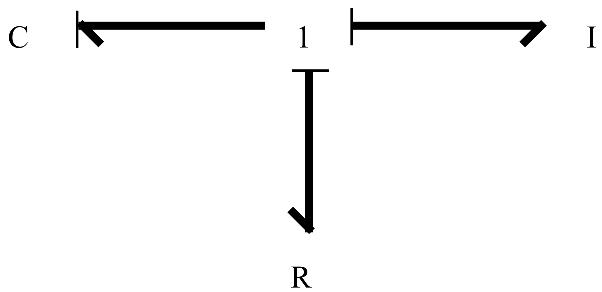
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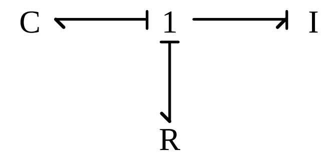
The three different ‘forms” of the model equations are distinct;

—they require different operators.



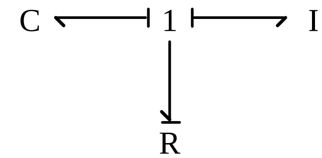
Version 1:

$v_m := \int dt \left(\frac{F_m}{m}\right)$ $\qquad F_m := -F_b - F_k$

$F_b := b\, v_b$ $v_b := v_m$

$F_k := \int dt \left(k\, v_k\right)$ $\qquad v_k := v_m$

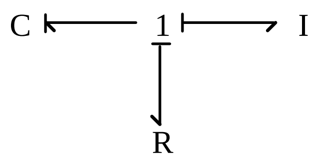
Version 2:



$$v_b := \frac{F_b}{b} \quad F_b := -F_m - F_k$$

$$F_m := m \frac{d}{dt} v_m \qquad v_m := v_b$$

$$F_k := \int dt \left(k\, v_k\right) \qquad v_k := v_b$$



Version 3:

$v_k := \frac{1}{k}\frac{d}{dt}\ F_k \quad F_k := -\ F_b - F_m$

$F_b := b\ v_b v_b := v_k$

$F_m := m\frac{d}{dt}\ v_m \qquad v_m := v_k$

In this simple example, the algebraic operators have inverses;

—that is not always the case.

THE (TIME-) INTEGRATION AND DIFFERENTIATION OPERATORS ARE *NOT* EQUIVALENT.

Integration tends to attenuate noise;

Differentiation tends to amplify noise.

Numerical integration tends to be stable;

Numerical differentiation tends to be unstable.

Mathematically:

The set of finite-valued but possibly discontinuous functions of time is closed under integration;

that set is not closed under differentiation.

Version 1 is preferable

It corresponds to a *state determined* representation.

For example, define a *state vector*

$$x = \begin{bmatrix} F_k \\ v_m \end{bmatrix}$$

and the system equations may be written in the form

$$\dot{x} := A\,x$$

as follows

$$\frac{d}{dt}\begin{bmatrix} F_k \\ v_m \end{bmatrix} := \begin{bmatrix} 0 & k \\ -\frac{1}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} F_k \\ v_m \end{bmatrix}$$

The (time-) integration operator is used to generate a state trajectory x(t) from an initial condition.

$$x(t) := \int_{t_o}^{t} A\,x(t)\,dt + x(t_o)$$

CAUSAL ANALYSIS

IDENTIFIES *INDEPENDENT* ENERGY STORAGE ELEMENTS

Independent energy storage elements yield state variables

Inertias with effort input require time integration to determine their flow output.

$$f(t) := \Psi\{p(t)\}$$

$$p(t) := \int_{t_o}^{t} e(t)\, dt + p(t_o)$$

Capacitors with flow input require time integration to determine their effort output.

$$e(t) := \Phi\{q(t)\}$$

$$q(t) := \int_{t_o}^{t} f(t)\, dt + e(t_o)$$

— **This is called *integral causality.***

CAUSAL ANALYSIS

IDENTIFIES *DEPENDENT* ENERGY STORAGE ELEMENTS

Inertias with flow input require time differentiation to determine their effort output.

Capacitors with effort input require time differentiation to determine their flow output.

- **This is *differential causality.***

(also called *derivative causality*.)

IDENTIFIES STATE VARIABLES

Each constant of integration that can be specified independently identifies a state variable.

State variables arise from energy storage elements.

Integral causal forms yield state variables.

- **Differential causal forms do not.**