2.092/2.093

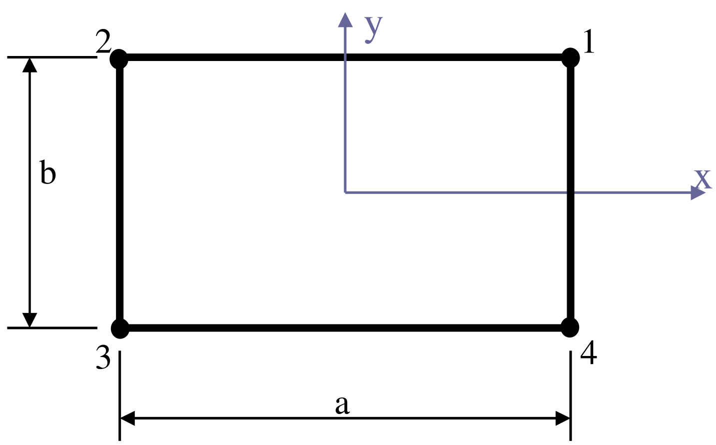
FINITE ELEMENT OF SOLIDS AND FLUIDS I

FALL 2009

Homework 5- solution

Instructor:	Prof. K. J. Bathe	Assigned:	Session10
TA:	Seounghyun Ham	Due:	Session12

Problem 1 (20 points):



$$h_1 = \frac{1}{4}\left(1+\frac{2x}{a}\right)\left(1+\frac{2y}{b}\right)$$

$$h_2 = \frac{1}{4}\left(1-\frac{2x}{a}\right)\left(1+\frac{2y}{b}\right)$$

$$h_3 = \frac{1}{4}\left(1-\frac{2x}{a}\right)\left(1-\frac{2y}{b}\right)$$

$$h_4 = \frac{1}{4}\left(1+\frac{2x}{a}\right)\left(1-\frac{2y}{b}\right)$$

$$u = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, v = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \begin{bmatrix} h_{1,x} & 0 & h_{2,x} & 0 & h_{3,x} & 0 & h_{4,x} & 0 \end{bmatrix} \underline{U}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \begin{bmatrix} 0 & h_{1,y} & 0 & h_{2,y} & 0 & h_{3,y} & 0 & h_{4,y} \end{bmatrix} \underline{U}$$

where $\underline{U^T = \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 \end{bmatrix}}$; $\varepsilon_{zz} = 0$

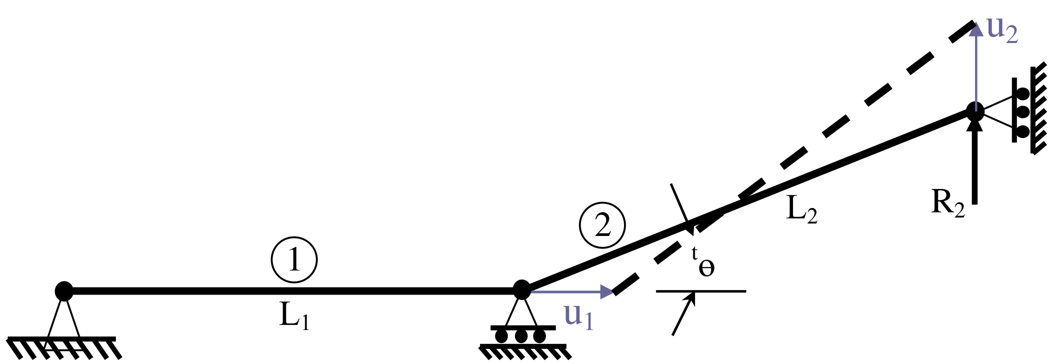
As $\varepsilon_V = \varepsilon_{xx} + \varepsilon_{yy}$

$$\therefore \varepsilon_V = \underline{B}_{\varepsilon V} \underline{U} = \begin{bmatrix} h_{1,x} & h_{1,y} & h_{2,x} & h_{2,y} & h_{3,x} & h_{3,y} & h_{4,x} & h_{4,y} \end{bmatrix} \underline{U}$$

Hence

$$\underline{\underline{K} = t \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \beta \underline{B}_{\varepsilon V}^T \underline{B}_{\varepsilon V} dxdy}$$

Problem 2 (20 points):



Assuming tension in the bar as positive, the equilibrium of the joints gives:

$$ {}^tP_1 - {}^tP_2 \cos {}^t\theta = 0 \qquad (1) \qquad R_2 - {}^tP_2 \sin {}^t\theta = 0 \qquad (2) $$

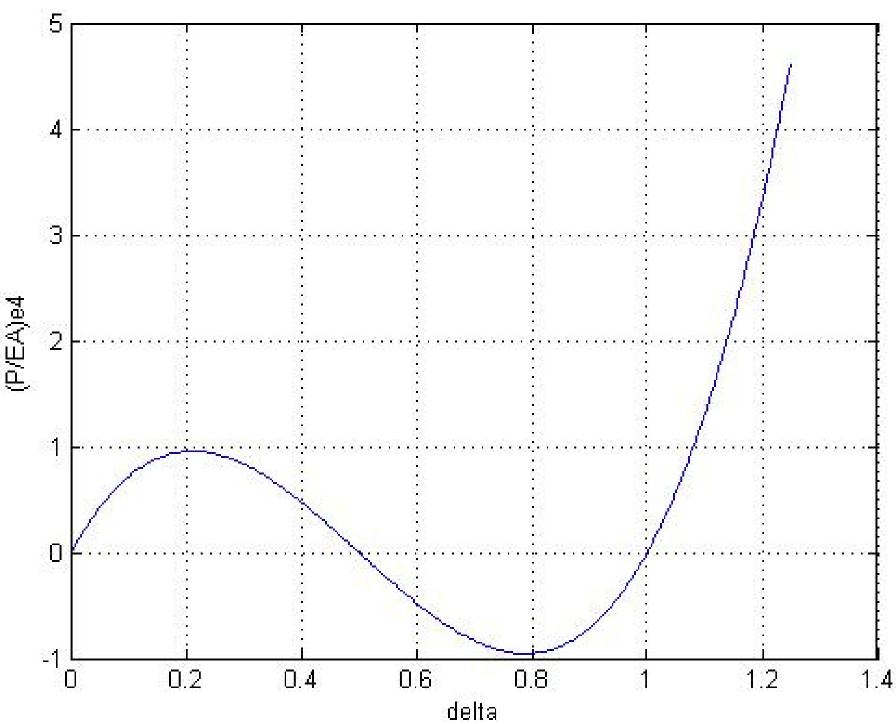
$$ {}^tP_1 = \frac{EA}{L_1} {}^tu_1 , \quad {}^tP_2 = \frac{EA}{L_2} \delta L_2 \qquad (3) $$

where $\delta L_2 = \sqrt{(5 - {}^tu_1)^2 + (0.5 + {}^tu_2)^2} - L_2 \qquad (4)$

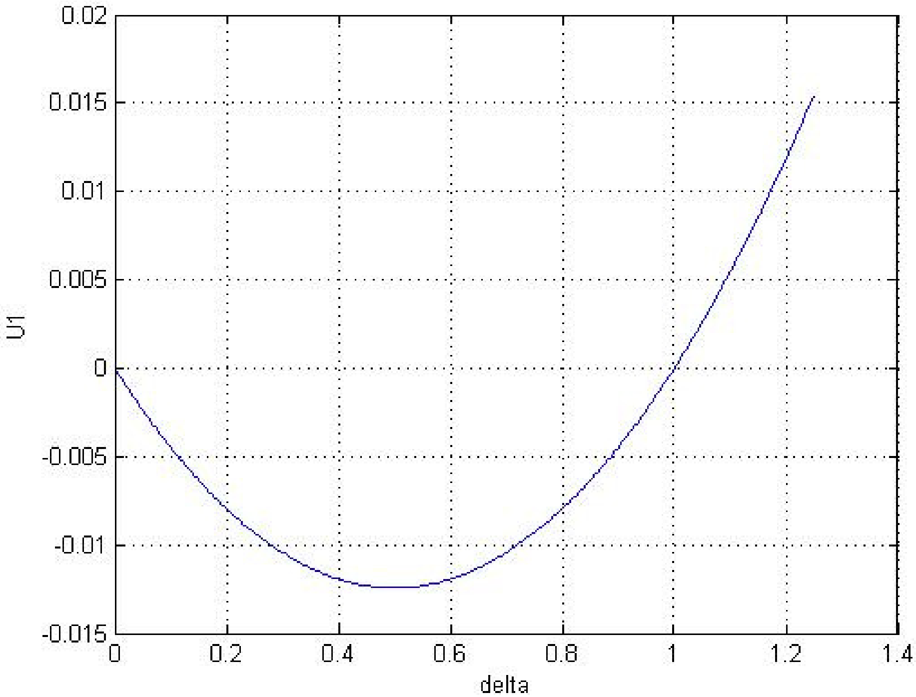
From the geometry, $\tan {}^t\theta = \frac{0.5 + {}^tu_2}{5 - {}^tu_1}$, ${}^tu_2 = -\Delta$, $R_2 = -P \quad (5)$

Eq. (1) and (2) are the force equilibrium equations. We use them by assuming a Δ, solving from equation (1) for ${}^tu_1$, then substituting ${}^tu_1$ and Δ into the equation (2) to obtain the corresponding P. We can also solve them in different way. We first assume ${}^t\theta$ and then calculate ${}^tu_1$ and Δ.

$$ \frac{P}{EA} \times 10^4 \text{ Vs. } \Delta $$



Displacement u_1 Vs. Δ



$$\frac{P}{EA}\times 10^4 \text{ Vs. } \Delta \text{ Using ADINA}$$