Yield Criteria

Ref: Shames section 7.5 and 9.2
or Crandall and Dahl section 5.11 page 312 ff

general state of stress => expressing maximum shear stress on octahedral plane closing in on a point = τ_{oct}

$$\tau_{oct} := \frac{1}{3}\cdot\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

where σ_1, σ_2, and σ_3 are principal stresses
($\tau_{12} = \tau_{13} = \tau_{23} = 0$)

onset of yielding occurs when τ_{oct} reaches a point depending only on the material. It can be evaluated from a tensile test where: $\sigma_1 = \sigma_y$; $\sigma_2 = 0$; $\sigma_3 = 0$; $\tau = 0$

$$\tau_{oct\_Y} := \frac{1}{3}\cdot\sqrt{(\sigma_y - 0)^2 + (\sigma_y - 0)^2 + (0 - 0)^2} \qquad \tau_{oct\_Y} := \frac{\sqrt{2}}{3}\cdot\sigma_Y$$

or ....

$$\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} = \sqrt{2}\cdot\sigma_Y$$

=> onset of yielding

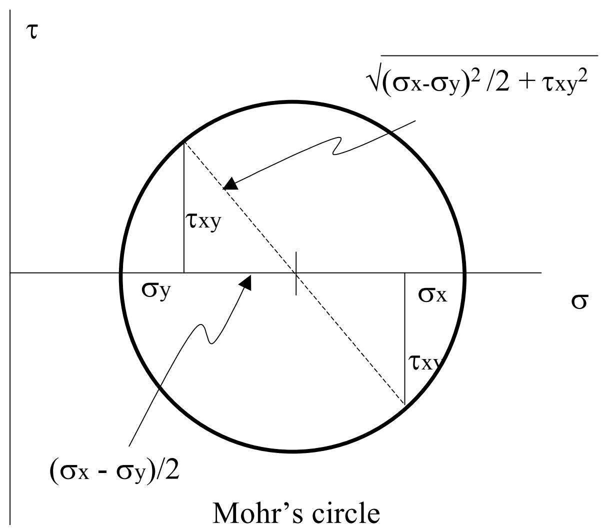
in 2D we have Mohr's circle (Shames 7.5):

$$\sigma_a := \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}} \qquad (3)$$

$$\sigma_b := \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}} \qquad (4)$$

$$\tau_{extreme} := \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

where +/- applies



Mohr's circle

if x and y are principal axes +> $\tau_{extreme} := \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2}$ and as above

$$\tau_{oct\_extreme} := \frac{1}{3}\cdot\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

where σ_1, σ_2, and σ_3 are principal stresses ($\tau_{12} = \tau_{13} = \tau_{23} = 0$)

and failure occurs when (rewriting): $\sigma_Y := \frac{\sqrt{2}}{2}\cdot\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$

or ... in 2D (plate): $\sigma_Y := \frac{\sqrt{2}}{2}\cdot\sqrt{(\sigma_1-\sigma_2)^2+\sigma_1{}^2+\sigma_2{}^2}$

using (3) and (4) above => $(\sigma_1-\sigma_2)^2+\sigma_1{}^2+\sigma_2{}^2 = \sigma_x{}^2+\sigma_y{}^2+(\sigma_x-\sigma_y)^2+6\cdot\tau_{xy}{}^2$ and we can

then say => $\sigma_Y := \sqrt{\frac{1}{2}\cdot\left[\sigma_x{}^2+\sigma_y{}^2+(\sigma_x-\sigma_y)^2+6\cdot\tau_{xy}{}^2\right]}$ => onset of failure.

we will designate the LHS σ_{VM}:

$$\sigma_{VM} := \sqrt{\frac{1}{2}\cdot\left[\sigma_x{}^2+\sigma_y{}^2+(\sigma_x-\sigma_y)^2\right]+3\cdot\tau_{xy}{}^2}$$

we will say that "failure" occurs when PSF * $\sigma_{VM} = \sigma_Y$

where PSF = partial safety factor

as an aside this can be generalized to 3D (see Crandahl and Dahl et al pg 316 eqn 5.24):

$$\sigma_{VM} := \sqrt{\frac{1}{2}\cdot\left[\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)\right]^2+(\sigma_y-\sigma_z)^2\right]+3\cdot\tau_{xy}{}^2+3\cdot\tau_{xz}{}^2+3\cdot\tau_{yz}{}^2}$$