## Lecture 5 - 2003 Twist closed sections

As this development would be almost identical to that of the open section, some of the development is simply repeated (copied) from the open section development. pure twist around center of rotation D => neither axial ( $\sigma$ ) nor bending forces (Mx, My) act on section

$$\int \sigma \, dA = N_{X} \qquad \int \tau \cdot h_{p} \, dA = \int q \cdot h_{p} \, ds = T_{p} \qquad \int \sigma \, dA = 0$$

$$\int \sigma \cdot y \, dA = -M.z \qquad \int \tau \cdot \cos(\alpha) \, dA = \int q \cdot \cos(\alpha) \, ds = V_{y} \qquad \int \sigma \cdot y \, dA = 0$$

$$\int \sigma \cdot z \, dA = M_{y} \qquad \int \tau \cdot \sin(\alpha) \, dA = \int q \cdot \sin(\alpha) \, ds = V_{z} \qquad \int \sigma \cdot z \, dA = 0$$

## a) equilibrium of wall element:

pure twist => .  $\xi = \eta = 0 =>$ 

$$\frac{\delta v}{\delta x} = \frac{\delta \psi}{\delta x} \cdot \cos(\alpha) + \frac{\delta \eta}{\delta x} \cdot \sin(\alpha) + h_p \cdot \frac{\delta \phi}{\delta x} \qquad \text{becomes} \qquad \frac{\delta v}{\delta x} = h_D \cdot \frac{\delta \phi}{\delta x}$$

## b) compatibility (shear strain)

$$\frac{d}{ds}u + \frac{d}{dx}v = \gamma$$
 here is first change, we cannot set  $\gamma = 0$  as we did in the open problem

$$= \sum_{\substack{d \\ ds}} \frac{d}{ds} u = \gamma \cdot -\left(\frac{d}{dx}v\right) = \sum_{\substack{d \\ ds}} \frac{d}{ds} u = \frac{\tau}{G} \cdot -h_D \cdot \frac{\delta \phi}{\delta x}$$

$$u = \int_0^s \frac{\tau}{G} ds - \frac{\delta \phi}{\delta x} \cdot \int h_D ds + u_0(x) dx$$

for open sections 
$$u = -\frac{\delta \phi}{\delta x} \cdot \int h_D ds + u_0(x) \text{ as } \gamma \text{ is small } => = 0$$

other assumptions: section shape remains etc. same

$$M_x = 2 \cdot q \cdot A$$
  $q := \frac{M_x}{2 \cdot A}$  and .....  $M_x := G \cdot J \cdot \frac{\delta \phi}{\delta x}$   $\Longrightarrow$   $q := \frac{G \cdot J}{2 \cdot A} \cdot \frac{\delta \phi}{\delta x}$ 

A in these relationships is the "swept area" i.e. per Shames; "total plane area vector of the area enclosed by the midline s." near 14.21

$$\int_{0}^{s} \frac{\tau}{G} \, ds = \frac{q}{G} \int_{0}^{s} \frac{1}{t} \, ds = \frac{G \cdot J \cdot \frac{\delta \Phi}{\delta x}}{2 \cdot A \cdot G} \cdot \int_{0}^{s} \frac{1}{t} \, ds = \frac{J}{2 \cdot A} \cdot \int_{0}^{s} \frac{1}{t} \, ds \cdot \frac{\delta \Phi}{\delta x}$$

<u>.</u>

 $J = \frac{4 \cdot A^2}{\int_0^b \frac{1}{t} ds}$  integral 0 to b => circular (all way around) defining J from 14.21 (Bredt's formula)

 $\int_{1}^{s}$ 

$$u = \int_0^s \frac{\tau}{G} \, ds - \left( \int_0^s h_D \, ds \right) \cdot \frac{\delta \phi}{\delta x} + u_0(x) = \left( \frac{J}{2 \cdot A} \cdot \int_0^s \frac{1}{t} \, ds - \int_0^s h_D \, ds \right) \cdot \frac{\delta \phi}{\delta x} + u_0(x)$$

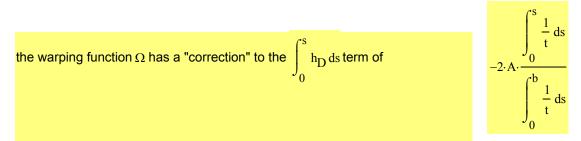
as with open sections **define** "sectorial" coordinate =  $\Omega$ , by its derivative  $\Omega$  wrt arbitrary origin and  $\omega$  wrt normalized sectorial coordinate

definition:

 $\int \frac{\tau}{G} ds$ 

$$d\Omega = \left(h_{D} - \frac{J}{2 \cdot A} \cdot \frac{1}{t}\right) \cdot ds = d\omega \qquad \Omega = \int_{0}^{s} h_{D} ds - \frac{J}{2 \cdot A} \cdot \int_{0}^{s} \frac{1}{t} ds = \int_{0}^{s} h_{D} ds - 2 \cdot A \cdot \frac{\int_{0}^{0} \frac{1}{t} ds}{\int_{0}^{b} \frac{1}{t} ds}$$

the warping function then becomes (as previously):  $u = -\frac{\delta \phi}{\delta x} \cdot \Omega + u_0(x) = -\phi' \cdot \Omega + u_0(x)$ 



otherwise everything is identical. hD and hc still have same meaning in  $\Omega$  and  $\omega$ 

b) warping stresses

as before: axial strain = du/dx =>  $u' = -\phi'' \cdot \Omega + u'_0(x)$  and

axial stress:

 $\sigma = E \cdot u' = -E \cdot \phi'' \cdot \Omega + E \cdot u'_{0}(x)$ 

$$\int \sigma \, dA = 0 \quad \text{determines } u'_0(x) \quad \int \left( -E \cdot \phi'' \cdot \Omega + E \cdot u'_0(x)' \right) dA = 0 \quad =>$$

$$E \cdot u'_{0}(x) = E \cdot \phi'' \frac{\int \Omega \, dA}{A} \quad \text{and stress becomes:} \quad \sigma = -E \cdot \phi'' \cdot \Omega + E \cdot \phi'' \cdot \frac{\int \Omega \, dA}{A} = -E \cdot \phi'' \cdot \omega$$
  
that is:  $\sigma = -E \cdot \phi'' \cdot \left(\Omega - \frac{\int \Omega \, dA}{A}\right) = -E \cdot \phi'' \cdot \omega \quad \text{where} \quad \omega = \Omega - \frac{\int \Omega \, dA}{A}$   
axial stress:  $\sigma = -E \cdot \phi'' \cdot \omega$ 

snear stress

shear flow follows from integration of  $\frac{d}{ds}q + \left(\frac{d}{dx}\sigma\right) t = 0$  along s and leads to :

$$\frac{d}{ds}q = -\left(\frac{d}{dx}\sigma\right) \qquad \Longrightarrow \qquad q(s,x) = -\int \frac{d}{dx}\sigma \, ds + q_1(x)$$

using the expression for axial stress  $\sigma = E \cdot u' = -E \cdot \phi'' \cdot \omega$ 

$$q(s,x) = q_1(x) - \int_0^s \left(\frac{d}{dx}\sigma\right) \cdot t \, ds = q_1(x) - \int_0^s -E \cdot \phi''' \cdot \omega \cdot t \, ds = q_1(x) + E \cdot \phi''' \left(\int_0^s \omega \cdot t \, ds\right)$$

where  $q_1(x)$  is f(x) unlike open section we cannot set it = 0  $q_1(x) \neq 0$ 

we can superpose an open and closed problem setting the "slip" i.e.  $\gamma$  at an arbitrary cut = 0 this is equivalent to collecting all the s variation into the open solution and the x variation into the constant

$$q_{\text{open}}(s,x) = \tau_{\text{open}} \cdot t = \frac{-T_{\omega}}{I_{\omega\omega}} \cdot Q_{\omega}$$
  $Q_{\omega} = \int \omega \, dA = \int_{0}^{s} \omega \cdot t \, ds$ 

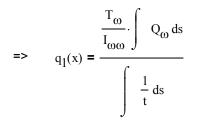
the  $\omega$  derived above is the value with the constant of integration set to zero, i.e starting from open end.

$$q(s,x) = q_1(x) + q_{open}(s,x) = q_1(x) - \frac{T_{\omega}}{I_{\omega\omega}} \cdot Q_{\omega}$$

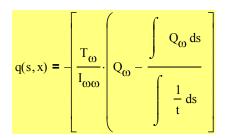
no slip =>  $\int \gamma \, ds = 0 = \int \frac{\tau}{G} \, ds = \int \frac{q}{t \cdot G} \, ds = 0$  N.B. these integrals are c i.e. no slip results are for complete way around the

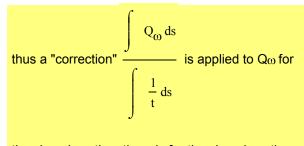
N.B. these integrals are circular complete way around the closed section

$$\Rightarrow \qquad 0 = \int \frac{q}{t} \, ds = \int \frac{q_1(x) - \frac{T_{\omega}}{I_{\omega\omega}} \cdot Q_{\omega}}{t} \, ds = q_1(x) \cdot \int \frac{1}{t} \, ds - \frac{T_{\omega}}{I_{\omega\omega}} \cdot \int Q_{\omega} \, ds$$



so we can say:





the closed section. the  $\omega$  is for the closed section (with it's correction applied)

## c) Center of twist

as for an open section, the second and third equilibrium condition above requires:

$$\int \sigma \cdot y \, dA = 0 \qquad \int \sigma \cdot z \, dA = 0 \qquad \text{for pure twist}$$
  
using  $\sigma = -E \cdot \phi'' \cdot \omega$  this requires  $\int \omega \cdot y \, dA = 0$  and  $\int \omega \cdot z \, dA = 0$  as  $E \neq 0$  and  $\phi'' \neq 0$ 

as shown above this relationship is identical with the new "corrrected"  $\omega$  so the shear center and center of twist can be calculated the same way.

$$y_{D} = \frac{\left(I_{y\omegac} \cdot I_{z} - I_{yz} \cdot I_{z\omegac}\right)}{\left(I_{y} \cdot I_{z} - I_{yz}^{2}\right)} \quad \text{and } \dots \qquad z_{D} = \frac{\left(-I_{z\omegac} \cdot I_{y} + I_{yz} \cdot I_{y\omegac}\right)}{\left(I_{y} \cdot I_{z} - I_{yz}^{2}\right)}$$

and for principal axes  $I_{yz} = 0$   $I_{yz} := 0$ 

$$\begin{split} \mathbf{y}_{D} &\coloneqq \frac{\left(\mathbf{I}_{y\omegac} \cdot \mathbf{I}_{z} - \mathbf{I}_{yz} \cdot \mathbf{I}_{z\omegac}\right)}{\left(\mathbf{I}_{y} \cdot \mathbf{I}_{z} - \mathbf{I}_{yz}^{2}\right)} \qquad \qquad \mathbf{z}_{D} \coloneqq \frac{\left(-\mathbf{I}_{z\omegac} \cdot \mathbf{I}_{y} + \mathbf{I}_{yz} \cdot \mathbf{I}_{y\omegac}\right)}{\left(\mathbf{I}_{y} \cdot \mathbf{I}_{z} - \mathbf{I}_{yz}^{2}\right)} \\ \mathbf{y}_{D} &\to \frac{\mathbf{I}_{y\omegac}}{\mathbf{I}_{y}} \qquad \text{and} \dots \qquad \qquad \mathbf{z}_{D} \to \frac{-\mathbf{I}_{z\omegac}}{\mathbf{I}_{z}} \end{split}$$