Problem Set No. 8

Out: Wednesday, November 10, 2004
Due: Wednesday, November 17, 2004 at the beginning of class

## Problem 1

Reconsider Problem 1 of Quiz No.1. As shown in the figure, the flywheel spins at a constant rate $\omega_{2}$, and also rotates about $z$ axis with angular velocity $\omega_{1}$ that is a function of time. The center of mass of the flywheel is located on the $z$ axis, and the centroidal moments of inertia are $I_{1}$ about the spin axis and $I_{2}$ transverse to that axis
(a) Derive the Lagrangian equations of motion.
(b) Find the generalized forces necessary to maintain the motion.


Problem 2 (from Doctoral Qualifying Exam 2002)
Two identical rigid cylinders, each having radius $R$ and mass $m$, are linked by a connecting rod of length $3 R$ and mass $M$ as shown below. A horizontal force $F(t)$ is applied to the center of the right cylinder and neither cylinder slips in its rolling motion. In the initial position, the angle $\theta$ locating the connecting pin is zero. Derive the Lagrangian equations of motion for this system.


Courtesy of Prof. T. Akylas. Used with permission.

## Problem 3 (from Ginsberg, Problem 6.41)

The slider, whose mass is $m_{1}$, oscillates within the groove in the housing. The moment of inertia of the housing about the axis of rotation is $I$. The spring restraining the slider is unstretched when $s=0$. Derive differential equations for the distance $s$ and spin angle $\phi$ resulting from application of a torque $M(t)$ to the shaft. Use the Lagrangian equations of motion.


Figure by OCW. After problem 6.41 in Ginsberg, J. H. Advanced Engineering Dynamics. 2nd ed. New York: Cambridge University Press, 1998.

Problem 4 (adapted from Ginsberg, Problem 6.42)

The bar, whose mass is $m$, is pinned to a collar that permits precessional rotation $\psi$ about the vertical guide, as well as nutational rotation $\theta$. The collar is fastened to a spring whose extensional stiffness is $k_{e}$ and whose torsional stiffness for precessional rotation is $k_{t}$.
(a) Derive the Lagrangian equations of motion for this system.
(b) Determine the constraint forces using Lagrange multipliers.


Figure by OCW. After problem 6.42 in Ginsberg, J. H. Advanced Engineering Dynamics. 2nd ed. New York: Cambridge University Press, 1998.

