10   Simulation of a System Driven by a Random Disturbance

1. Simulate the second-order system:

$$x'' + ax' + bx \;=\; d(t)$$

   with $a = 0.4$ and $b = 2.25$.

   Figure 1 below shows responses to the same disturbance $d(t)$ for all the three values of a. Note that because the phases are generated from random numbers, your figure might not look like this, although the size of the response should be similar. Also, the component periods repeat within sixty seconds, so $d(t)$ repeats itself in this graph.

2. From the graph, about what is the "significant height" of the motion?

   The significant height of the response with $a = 0.4$ is around two units.

3. What is the effect of reducing or increasing the damping in this system, say $a = 0.2$ and then $a = 1.0$?

   The response of the system is clearly strongly dependent on the damping ratio: the oscillations are much larger as the damping ratio gets smaller. This is no surprise since the undamped natural frequency is 1.5, near where the disturbance frequencies lie. This example shows a rather bad resonance condition between the physical system and the disturbance, which can be mitigated by increasing the damping.

   With regard to design, however, it is worth pointing out that if the damping increases, then the system responds more slowly to initial conditions and to impulsive disturbances. In other words, if a boat or airplane is knocked over by a big wave or wind gust, damping will slow down the return to upright. Most people don't like that!

```
%-------------------------------------------------------------------------
% Time-domain simulation of a second-order system with two-mode
% random disturbance.

clear all;
global a b omega A phi ;

% set the frequencies, amplitudes, and phase angles
omega = .5:.25:2.5 ;
A = [.2 .4 .6 .2 .1 .4 .3 .2 .1] ;
phi = 2*pi*rand(size(A)) ;

Tfinal = 60 ; % final time
```
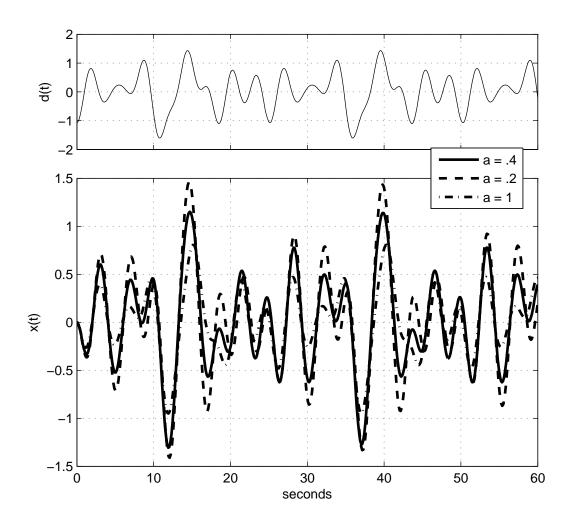
Figure 1: Time-domain simulation of vehicle subject to random disturbance.

```
b = 2.25 ;  % wn^2; spring term

% run the simulation for three values of a

a = .4 ;
[t1,y1] = ode45('td_veh_dxdt',[0 Tfinal],[0 0]) ;

a = .2 ;
[t2,y2] = ode45('td_veh_dxdt',[0 Tfinal],[0 0]) ;

a = 1 ;
[t3,y3] = ode45('td_veh_dxdt',[0 Tfinal],[0 0]) ;

% recreate d(t) here in the main program
```

```
tvec = 0:.1:Tfinal;
d = zeros(1,length(tvec));
for i = 1:length(omega),
    d = d + A(i) * sin(omega(i)*tvec + phi(i)) ;
end;

% plot the results

figure(1);clf;hold off;
subplot('Position',[.2 .75 .6 .2]);
plot(tvec,d);
ylabel('d(t)');
grid;
set(gca,'XTickLabel',[]);

subplot('Position',[.2 .2 .6 .5]);
plot(t1,y1(:,2),'-',t2,y2(:,2),'--',t3,y3(:,2),'-.','LineWidth',2);
legend('a = .4', 'a = .2', 'a = 1',3);
grid;
xlabel('seconds');
ylabel('x(t)');

disp('Move label if necessary, then hit a key to save.');
pause ;

print -deps td_veh.eps
%-----------------------------------------------------------------


%-----------------------------------------------------------------
% Time Domain simulation of second-order system - derivative
% function

function [xdot] = td_veh_dxdt(t,x) ;

global a b A omega phi ;

% construct the disturbance - it is purely a function of time
d = 0 ;
for i = 1:length(omega),
    d = d + A(i) * sin(omega(i)*t + phi(i)) ;
end;

% calculate the derivatives (column!)
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```
xdot(1,1) = -a*x(1) - b*x(2) + d ;
xdot(2,1) = x(1) ;
%-------------------------------------------------------------
```

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