Massachusetts Institute of Technology
2.008: Design and Manufacturing II
Professor Jung-Hoon Chun

Problem Set 7

Problem 7-1. Systems

(a) Explain what happens to inventory as cycle time decreases.

Solution: Recall that Little's law states that $L = \lambda w$ where L is the average inventory, λ is the average production rate and w is the cycle time. Therefore, as cycle time decreases, the average inventory should also decrease.

(b) Describe your process of doing laundry. How many operations, what are the respective production rates, do you have any buffers, what is the cycle time?

Solution: The laundry piles up into big loads. I curse the laundry at a rate of 10^6 utterances per minute. I then vacuum seal the laundry into a container at a rate of 1 pound per second, and then deliver it to a service (3 minutes) down the street, and then pay a fixed rate per pound to have it done. The cycle time is 24 hours.

Problem 7-2. Boston Weather

(a) The average daily temperature for Boston in May is $58.5°F$. Suppose the standard deviation is $8.157°F$ and assume the temperature obeys a normal distribution. How many days in May are less than $43°F$? (This is a bad analogy since we are making assumptions that are totally bogus. Just be sure that you can do simple calculations with normal distributions for the exam.)

Solution: We would like to know the number of days that fall $(58.5 - 43)/8.157 = 1.9$ standard deviations outside of the mean in one direction (ie, we dont care about the days that are $58.5 + (1.9 * 8.157)$ degrees). Looking at our table, we see that for $z = 1.9$, the area under the normal distribution curve is 0.9713. Therefore $(1 - 0.9713)31 = 0.88$ days in May are below $43°F$.

Boston May Temp (F) ———

$30°$　　$43°$　　$58.5°62°$　　　　　$90°$
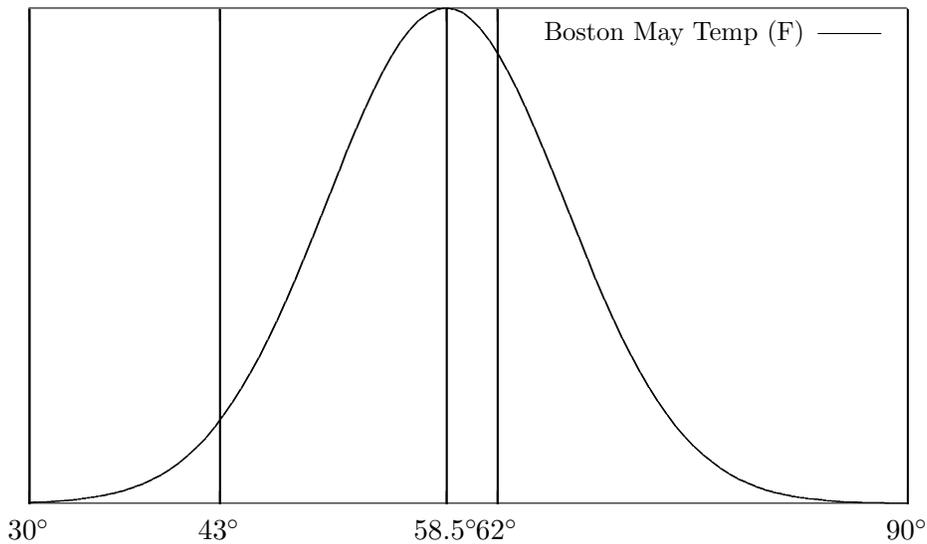
(b) As a condition of coming to MIT for graduate school, I sent MIT my specifications for the temperature range during the month of May (MIT controls everything, dont they?) In particular, I agreed to a daily temperature of $62°F$ but would tolerate plus or minus 5 degree shifts. Based on the data from (a), what is MIT's capability index (C_{pk})?

　　Solution:　This involves plugging into the formulas for C_{pk}:

$$
\begin{aligned}
Z_{min} &= \frac{|(62-5)-58.5|}{8.157} = -0.123 \\
Z_{max} &= \frac{(62+5)-58.5}{8.157} = 1.04 \\
C_{pk} &= Z_{min}/3 = 0.04
\end{aligned}
$$

Problem 7-3. Shaft

(a) Your enterprise is using a process which produces parts whose length have a mean 2.5in and standard deviation 0.015in. You plan to make 100,000 of these parts. You have determined by taking into account the rework cost and the loss of customer satisfaction that the quality loss function for the process is:

$$L = 2250\sigma^2$$

where σ is the standard deviation of length of the part and L is the average quality lost in dollars per part.

A very smart but somewhat expensive Harvard graduate has offered you her expert advice on your process. For every 0.001in reduction in the standard deviation of the length of the part, she will charge 6000.

How much of her advice do you buy? (You can buy fractional amounts of her consulting).

Solution: Let $L(x)$ represent quality lost as a function of the amount of advice that you buy. We can easily express $L(x)$ and then minimize it by taking the first derivative and setting it to zero.

$$
\begin{aligned}
L(x) &= (10^5)2250(0.015 - 0.001x)^2 + 6000x \\
&= 225x^2 - 750x + 50625 \\
L'(x) &= 450x - 750 \\
&\quad \text{Setting this to zero, we get that} \\
x^* &= 1.66
\end{aligned}
$$

It pays to buy 1.66 units of consulting.