2.007 Design and Manufacturing I
Spring 2009

<u>**2.007 Design and Manufacturing 1**</u>

Homework #3 Pneumatics, CAD, & Gears

SOLUTIONS

1) (25 points)

Note from Professor Frey:

The questions on the survey are taken from a body of test material in physics known as the Force Concept Inventory. In order to preserve the value of the test for future administrations of the same test, I can't distribute detailed solutions to the Force Concept Inventory. Suffice it to say, the questions are all based on Newton's three laws of motion.

First Law: A body persists its state of rest or of uniform motion unless acted upon by an external unbalanced force.

Second Law: Observed from an inertial reference frame, the net force on a particle of constant mass is proportional to the time rate of change of its linear momentum: F = d(mv)/dt. This law is often stated as, "Force equals mass times acceleration (F = ma)."

Third Law: Whenever a particle or body exerts a force on another particle or body, that second body simultaneously exerts a force the other body with the same magnitude in the opposite direction. "To every action there is an equal and opposite reaction."

2)

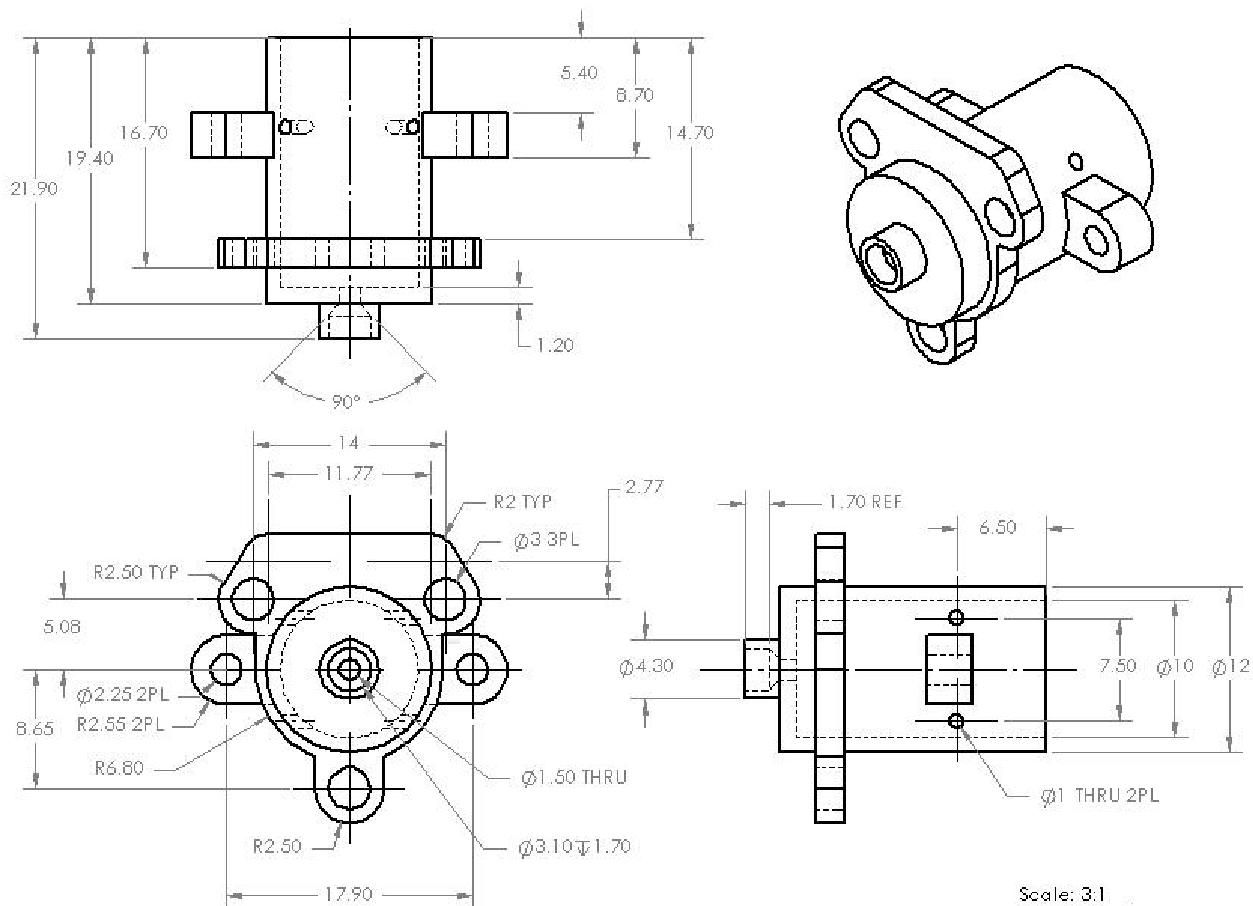
a)

See attached drawing. Making a good engineering drawing takes a lot of time and *attention to detail*. It's not as simple as putting in dimensions, although as a first criterion the drawing must have *all* the dimensions required to create the part. Even a single missing or ambiguous dimension will put your part on hold in the shop.

There are also conventions and good practices which were introduced during the CAD lectures. Some common mistakes based on a random sampling of submissions:

- Use third angle projection, align views, and show hidden lines in all three views.
- Don't put dimensions inside the part. Use extensions to bring all dimensions to the outside of the views. If necessary, reduce the scale of the drawing to leave more room for dimensions.
- Clearly indicate center lines and center marks for holes. Assumptions of symmetry can only be made if the centerlines are clearly marked.

- Don’t forget to dimension fillet radii and centers. This is usually more useful than dimensioning the length of tangent lines between fillets.
- Use the views to your advantage. Don’t load all the dimensions into a single view. Use a scheme that makes sense, such as dimensioning all the features to be milled in the front view. Keep the machining process in mind.
- Indicate critical dimensions. The distance between two holes might be more important than the distance between one hole and the edge of the part. Leave non-critical dimensions to be “driven.” This requires you to know the functionality of the part, of course.

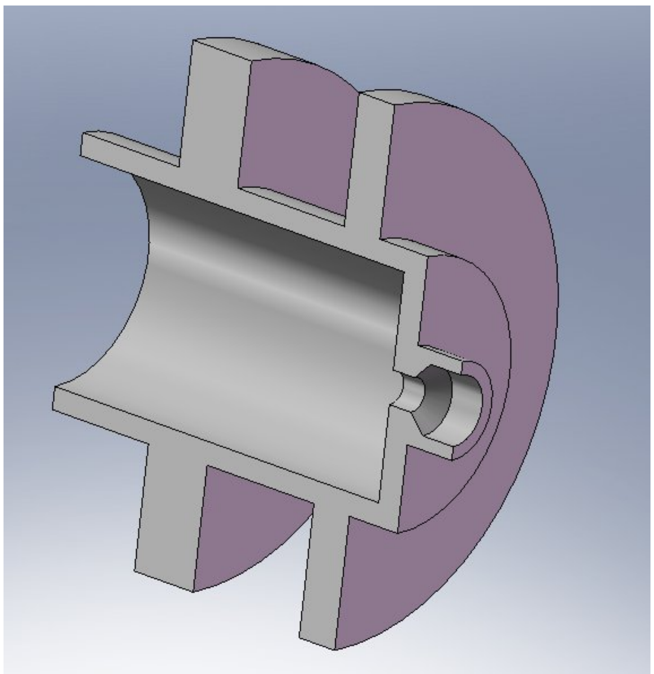


Scale: 3:1
Dimensions in mm.

b)

There are obviously many good answers to this part. The consensus seemed to be that there were both milling and turning/facing operations required. The order in which you do these operations is important. The “favorite” order of operations was:

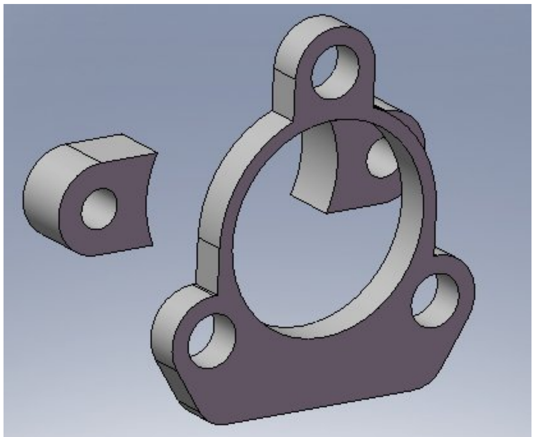
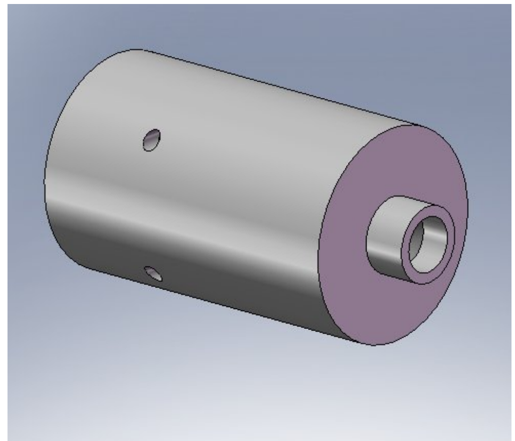
1. Turn and face, drill, and bore (in that order) the cylindrical stock into a shape looking roughly like this (half section view):

   Which actually requires you to flip the piece in the lathe to get access to both sides.
2. Mill the two-dimensional details of the flanges, including the holes. This also requires you to flip the part once in the mill to get access to both flanges. The fillets can be programmed into a CNC mill.
3. Drill the two small 1mm cross-holes. This will be hard since you are drilling into a curved surface. Need to use a stiff center drill first, as a few people correctly noted.

Variations on this including starting with rectangular stock and drilling the cross-holes first, a solution that I think makes sense, or doing things in a slightly different order. Milling the large bore was another common choice, although you would have to describe a good centering procedure.

The other common solution, which some listed as an improvement option, was to make this out of several parts:

 and 

The procedure includes milling the flat pieces and turning/facing/drilling/boring the cylindrical piece, then assembling them by various means. Cutting the flat pieces on the water jet is an option, but you will have some taper on the edges.

Solutions which have the two small flanges integrated into a single rectangular piece which fits into rectangular holes in the main cylinder will lose some credit: You can't easily make the rectangular holes and this does not exactly correspond to the model.

Ways to improve the part or fabrication were equally varied. Simplifying the geometries, separating the parts, using rapid-prototyping capabilities are all good ideas. Credit given for most reasonable proposals!

3)

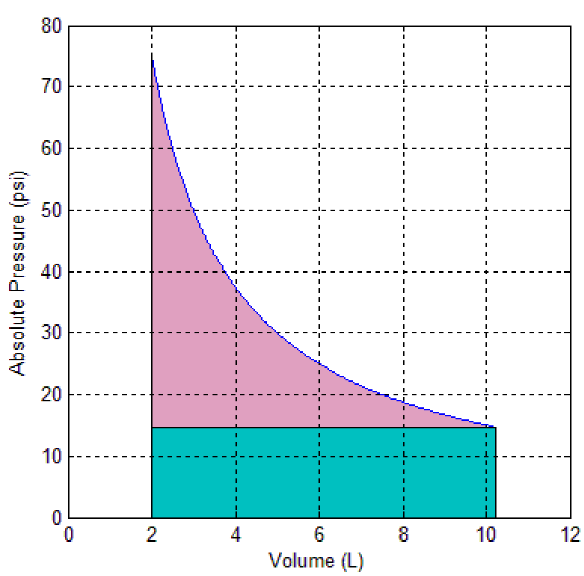
Credit goes to Michael Roberts and Rachel Batzer for the basics of this solution. Most were close on this, maybe not accounting for work done by/on the atmosphere or not using gauge pressure appropriately. Substantial partial credit will be given in most cases.

a)

$$P_1 = P_{atm} = 1.01 \times 10^5 \, Pa$$
$$P_2 = 60 \, psig = (4.14 + 1.01) \times 10^5 \, Pa = 5.15 \times 10^5 \, Pa$$
$$V_2 = 2L = 0.002 m^3$$
$$V_1 = \frac{P_2 V_2}{P_1} = 10.2 L = 0.0102 m^3$$

Integral of gauge pressure has two parts: work done total and work done by atmosphere. The net is the work we need to do to compress the gas.

$$W = \int_{V_1}^{V_2} \left[P(V) - P_{atm} \right] \cdot dV$$
$$W = P_1 V_1 \int_{V_1}^{V_2} \frac{1}{V} \cdot dV - \int_{V_1}^{V_2} P_{atm} \cdot dV$$
$$W = P_1 V_1 \left[\ln(V) \right]_{V_1}^{V_2} - P_{atm} \left[V \right]_{V_1}^{V_2}$$
$$W = P_1 V_1 \ln\left(\frac{V_2}{V_1} \right) - P_{atm} \left(V_2 - V_1 \right)$$
$$W = -1678 J + 828 J = -850 J$$



b)

To find the new initial volume:

$$PV^{1.4} = K$$
$$\Rightarrow V_1 = \left(\frac{P_2 V_2^{1.4}}{P_1}\right)^{\frac{1}{1.4}}$$
$$V_1 = 6.4L$$

The ideal gas law still holds:

$$PV = mRT$$
$$\Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$
$$\Rightarrow T_2 = T_1 \frac{P_2 V_2}{P_1 V_1}$$
$$T_2 = 467K = 194^{o} C$$

c)

$$F = PA \approx (60\,psig)\pi(0.5in)^2$$
$$F \approx 47lbf = 209N$$

Assuming the pressure drop due to expansion is negligible and steady-state (no losses in tubing/valves). Slightly lower if an adiabatic or isothermal expansion is assumed.

d)

Same procedure as part a but with a different function for P(V):

$$P_2 = P_{atm} = 1.01 \times 10^5\,Pa$$
$$P_1 = 60\,psig = (4.14 + 1.01) \times 10^5\,Pa = 5.15 \times 10^5\,Pa$$
$$V_1 = 2L = 0.002m^3$$
$$V_2 = \left(\frac{P_1 V_1^{1.4}}{P_2}\right)^{\frac{1}{1.4}} = 6.4L = 0.0064m^3$$

Integral of gauge pressure has two parts: work done total and work done on the atmosphere. The net is the work useful work done by the expanding gas.

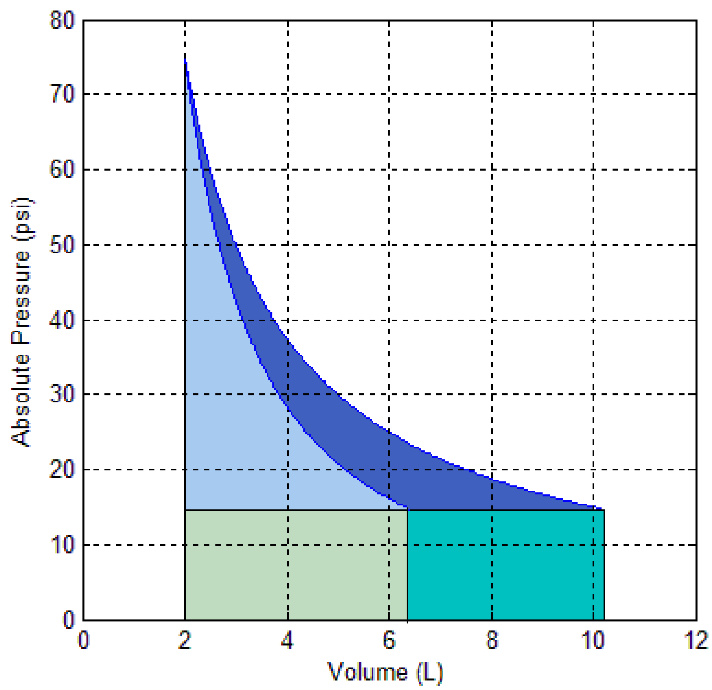
$$W = \int_{V_1}^{V_2} \left[P(V) - P_{atm} \right] \cdot dV$$

$$W = P_1 V_1^{1.4} \int_{V_1}^{V_2} \frac{1}{V^{1.4}} \cdot dV - \int_{V_1}^{V_2} P_{atm} \cdot dV$$

$$W = P_1 V_1^{1.4} \left[-\frac{1}{0.4} V^{-0,4} \right]_{V_1}^{V_2} - P_{atm} \left[V \right]_{V_1}^{V_2}$$

$$W = -\frac{P_1 V_1^{1.4}}{0.4} \left(V_2^{-0.4} - V_1^{-0.4} \right) - P_{atm} \left(V_2 - V_1 \right)$$

$$W = -958J + 445J = 513J$$

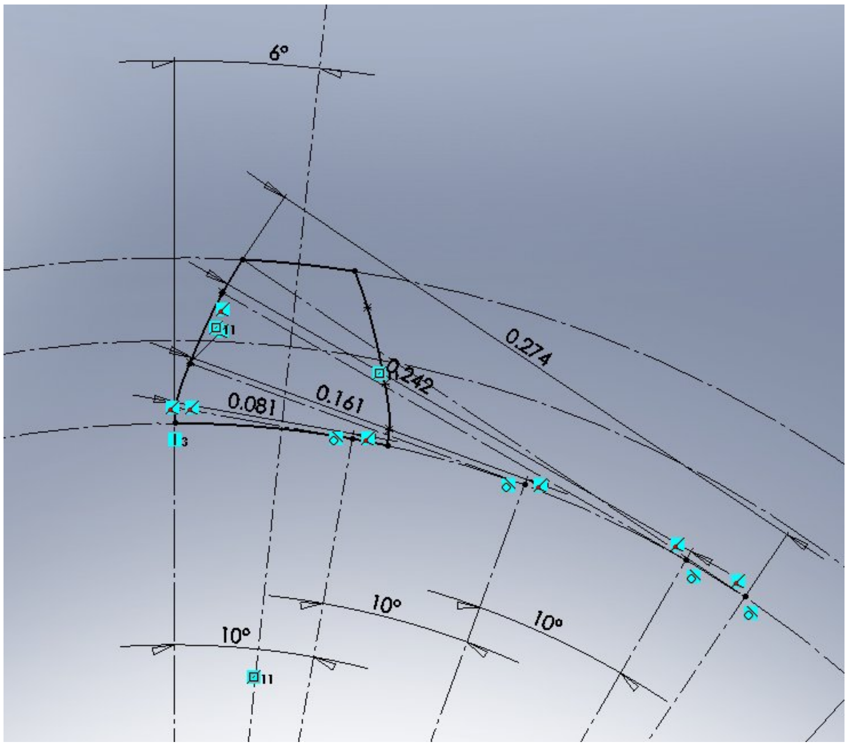


This shows the relevant integral components for both 3a and 3d. The adiabatic curve does less work than the isothermal curve puts in, as expected. The efficiency in this case is about 60%.

4)

a)

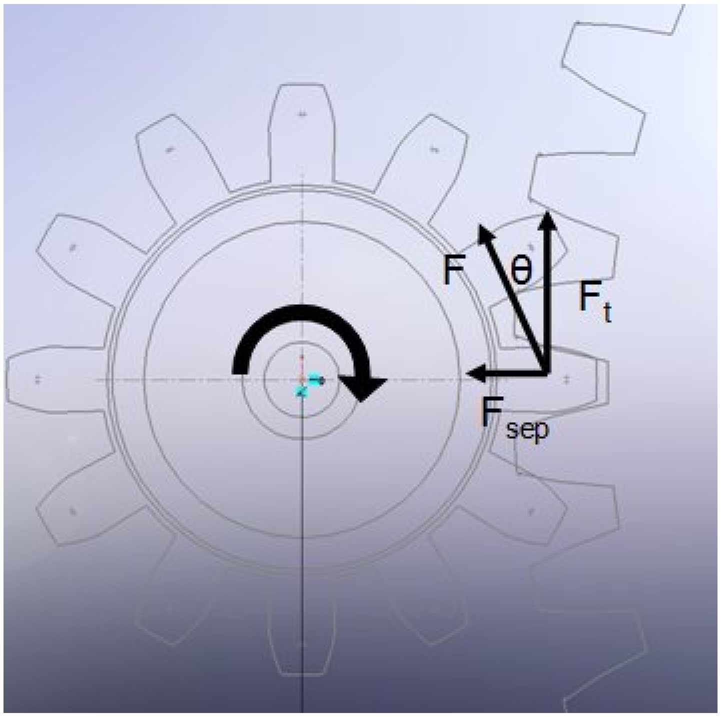
Here is the construction of a single gear tooth:



If you showed this construction, using arc lengths to generate the shape from the base circle, either in SolidWorks or by hand, that was worth full credit. Simple extruding the provided drawing into a 3D part was not the intent of part a), but given the server issues substantial credit will be given for just about anything that looks like a 24-tooth gear.

b)

Here, use of the available drawings is perfectly acceptable. For example:

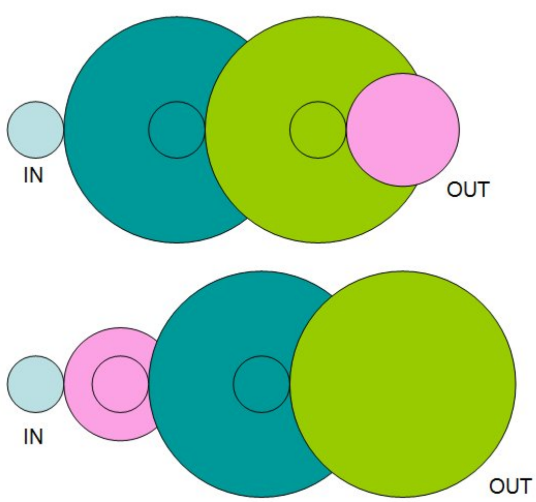


The true force between the gears acts along the pressure line. The tangential force is one component of this true force. The separation force is the other component. Then, the calculation of the separation force goes like:

$$F_{sep} = F_t \tan(\theta) = \frac{\tau}{r_{12}} \tan(\theta) = \frac{1Nm}{0.00635m} \tan(20^{\circ}) = 57N$$

c)

This can be done with a compound gear train. The most direct solution is 12:48, 12:48, and either 12:24 or 24:48. Just a couple examples:

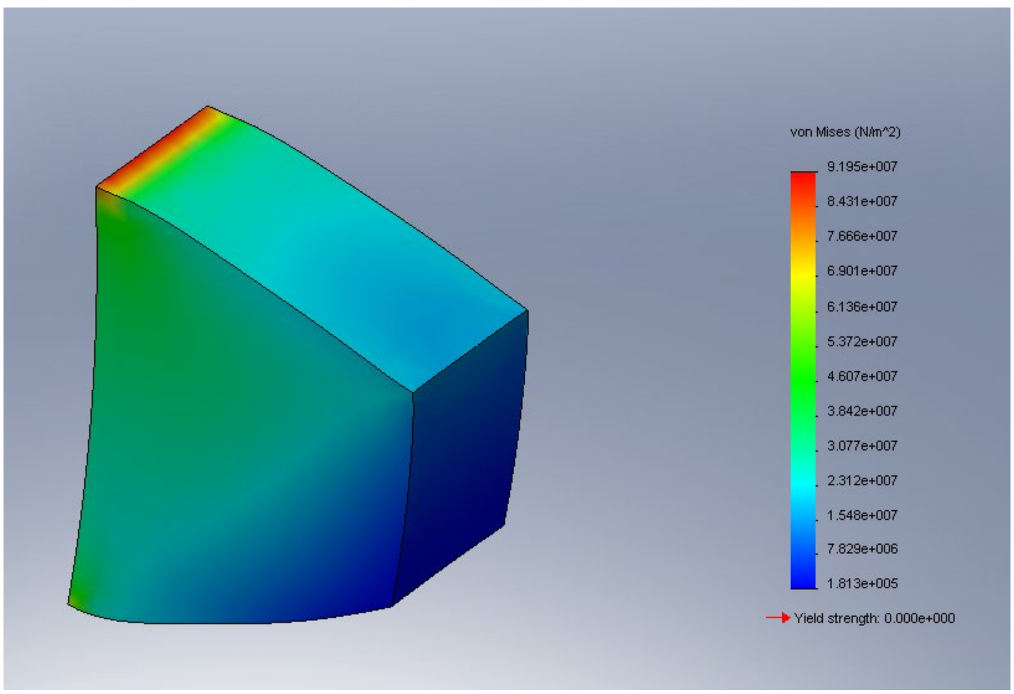


The latter is probably easier to build since all the gears can be supported by bearings on both sides. (The final output gear of the former cannot.) There are obviously many more ways of arranging this.

d)

Again, there are many ways of approaching this problem. The web tool, SolidWorks, or simple beam bending equations can all get to a good estimate. The steps involved are:

1. Determine the force on each gear in your specific gear train solution from part c). The forces will generally increase with each compound gear set, so the output gear will probably see the highest force.
2. Since the pitch is the same for all gears, the shape of the teeth will be roughly the same. It might make sense, then, to analyze the stress on the last stage of the gear train. It will see 32 times the stall torque, or just about 2Nm. If it is a 48-tooth gear, that will act at a radius of 1in or 0.0254m. This gives a force of 78N. Applying this load in COSMOS to a Delrin-like material:



This is just a very rough approximation, since the load won't be evenly distributed and SolidWorks doesn't even have a Delrin model. But it shows a maximum stress of 92MPa at the tooth base. This is greater than the yield stress of Delrin. The factor of safety in this case would be below one, meaning the gear is likely to fail. Your exact calculation will depend on your gear train design, but many would likely fail under the stall torque of the servo.