

MITOCW | R5. Equations of Motion

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J. KIM VANDIVER: We're waiting for people to come in. And what are the important concepts this week? There's only the one lecture, and the lecture yesterday was pretty dense in concepts. Let's build a quick list here. Who's got the first offering?

AUDIENCE: Rotational dynamics of rigid bodies.

J. KIM VANDIVER: OK, that was a general overall topic, so how about a concept though.

AUDIENCE: The mass moment of inertia matrix.

J. KIM VANDIVER: OK, yep, we introduced the concept of the mass moment of inertia matrix. And I'll give it a context, so you can, for example, write H as I times ω . So we introduced that, didn't get very far with it. Another one from yesterday?

AUDIENCE: I'm not sure if this fits but talking about how the body can be rotating around the point opposite the [INAUDIBLE].

J. KIM VANDIVER: Ah yeah. Yep So we talked about rotation. The subject yesterday was really rigid body rotation about fixed point on or off the body. So here's our axis of rotation. Call it A . And it's spinning about that point at some ω . But that point A could be over here too. And then the whole body is just going, so an example of rigid body rotation about a point not on the body would be something doing that.

OK, something else.

AUDIENCE: The angular velocity [INAUDIBLE].

J. KIM VANDIVER: Yes. Yeah, what does it mean when H and ω may not align? What's that mean? So we got into that a bit yesterday. And anything else that struck you?

OK, well that's a good start. Let's just work with that. Now I will do that second. We're going to come work a little bit, talking about this system in those contexts. Before I do that, I want to-- the staff decided we hadn't done something in lecture yet very carefully so we wanted to do it once carefully within recitation. And that's a methodical way of doing free body diagrams.

So we have a system, two masses, springs, dashpots. In the last class, there were a few people who didn't know really what we meant by k 's and c 's and how they work. So I will give you a quick definition.

So here we have a spring. And these are called linear springs, so Hooke's law. If you cause that spring to move, and I'll call it here the plus x direction-- you grab it and you pull on it-- you have to apply a force. So the force to pull that spring is positive kx in the same direction as x .

But if that spring's attached to a mass what direction is the force that the spring applies on the mass? So I've given you a hint here. It pulls back, right? Because this is Newton's third law. So we are doing free body diagrams, and we're concerned with forces on the rigid bodies.

So, this spring, if this mass went that way, the spring would pull this way. And we indicated the direction with an arrow and its value as a kx . Dashpots are similar. Except dashpots are activated by velocity. So this is a linear dashpot, such that it has a dashpot constant c . And if I give this a positive value of velocity, the force required to pull that dashpot is $c\dot{x}$. It's linearly proportional. Velocity and the force that if it were attached to a mass that the dashpot puts on the mass would be opposite direction but same value, $c\dot{x}$, but is resisting the motion. OK.

So I'm going to just give you general method for doing free body diagrams. And the reason we choose this problem is there's two bodies, and depending on which is doing what, this intermediate spring in here particularly can push or pull on either body and having a method so you don't mess up the signs is what this is all about.

So let's just start by building a free body diagram of mass one. And the assignment

I'm going to give you to work on, just seat work here for a minute, is only consider the springs for now. We'll just deal with the springs.

So assume the system is moving. You have to do free body diagrams of it. Put the spring forces on the mass in your free body diagrams. So spend a minute or so. Talk to your neighbors after you get a little ways along with it. And I'm going to add-- there's also on this whole system external force F_1 , external force F_2 . That's your excitation. But don't worry about that for now. Just figure out the spring forces, and I'll be quiet.

I missed a step, my apologies. We should do something else first. The very first step is to assign the coordinates. Let's all agree on the coordinate system here. So the question really is the first thing you have to do is you have to decide how many independent coordinates you need to completely describe the motion. So how many coordinates do you think are needed for this problem? I see a one.

AUDIENCE: I was just--

J. KIM VANDIVER: That's a second one. Anybody else have a suggestion? So where would you put the one?

AUDIENCE: The x term.

J. KIM VANDIVER: And measuring the displacement of mass one?

AUDIENCE: Sure.

J. KIM VANDIVER: OK. And is that all we need? Anybody have a further--

AUDIENCE: Oh, there's a displacement at mass two.

J. KIM VANDIVER: And it's independence, right? They can move-- how many coordinates now do you think you need? Uh oh. She changed her mind to two. And I've got them labeled up here for you already.

So it's going to take two. And one of the ways you can test if you have the right

number, if you think it's one, assign it. Freeze it. Is there still independent movement of a part of the system? If there is, you have yet another degree of freedom. If we freeze both x_1 and x_2 , and can anything move in the system? Nope. So we've got it nailed down. All right.

But where do you measure? There is something else you have to determine. And that is, where do you make zero? You also have to know that. So where is x is 0 in your picture? We'll just talk about both of them, x_1 and x_2 . How would you pick the 0 point? Christina?

AUDIENCE: Probably when the spring's at equilibrium so when it's not exerting force.
[INAUDIBLE].

J. KIM VANDIVER: OK, she says when there's not exerting force in either direction. I would call that the static equilibrium position. There is a conceptual problem with that. We'll assume that that will work if you've set the system up so that there's no forces in the springs. But you know, if I move this wall in a little bit and let it reach static equilibrium, it has a static equilibrium position. All the springs have forces in them.

Now, you pre-compress them. So there's kind of a problem-- you could run into a conceptual problem with that. It will still work. Picking a static equilibrium position is a good thing to do usually. OK.

There are other positions. For example, this system, it's a simpler system, just an ordinary little mass spring. But where do you make zero when you pick your coordinate?

AUDIENCE: In this case, you could zero it out at the rest length of the spring plus the length of this spring as a result of the force of gravity.

J. KIM VANDIVER: So right now you're seeing the static equilibrium position. The spring has tension in it, the weight of the mass, right? So you could either make x right here. Or the other obvious choice would be?

AUDIENCE: Where you're holding it.

AUDIENCE: You're holding it.

J. KIM VANDIVER: I heard a couple voice but I couldn't hear what you said.

AUDIENCE: Where you're holding it?

J. KIM VANDIVER: Well, yeah, but it turns out that actually is a terrible choice. Because then you have to get involved with the length of the spring. And the length of the spring actually really doesn't enter into the equation in motion unless you're really silly about where you pick the measure from. Static equilibrium is good, but the other one is the zero spring force position is the other natural one to do. So at zero spring force, that's no extension of the spring. Because this is preloaded when you do this, and have to figure out what that does to you. OK.

Again, static equilibrium has some real advantages here, but we'll talk about that later. Yeah?

AUDIENCE: So why does zero spring force work there but not--

J. KIM VANDIVER: I'm just saying there are actually situations where there is no zero force.

AUDIENCE: Oh, OK.

J. KIM VANDIVER: And you have a conception-- if you hadn't ever thought of that before, that could give you pause when you go to solve the problem. The springs are preloaded. Are your equations of motion valid?

So you preloaded this a little bit, squeezed it in, this system has natural frequencies, right? Do you agree with that? It's very similar to this system except it's just now-- it doesn't happen to have a third spring and have gravity involved. But this system has two natural frequencies, that one and one that's a little harder for me to do but that one.

So if I've preloaded this a little bit, squeezed it in some, would the natural frequencies change? Yes-- how many think yes? Get your hands up. Come on. You're gamblers. How many think yes? How many think no? How many just don't

want to raise their hand? [LAUGHS] Like you. OK.

The natural frequency doesn't change. It turns out to preload doesn't matter in a linear system like this. So there's some nuances in here. A simple system like this has some little traps in it. It will take them time to learn your way through.

But let's keep it simple. No preload, the static equilibrium position, draw the free body diagram for mass one, springs only. Only deal with the springs. Take a couple minutes and talk to your neighbor if you need to.

OK, so I won't come look at each of your things. I want to have different groups kind of-- they should take one force at a time. So you guys, give me a force on this mass, due to a spring. Give me a spring force and tell me what direction it's in.

AUDIENCE: I guess it's [INAUDIBLE].

J. KIM VANDIVER: So tell me which spring we're talking about.

AUDIENCE: So [? I didn't hear. ?] So is this being impressed, like are these springs?

J. KIM VANDIVER: No, this is static equilibrium position. But now if you give it initial [INAUDIBLE] and let go, it's going to sit there and do something. We're trying to derive the equations of motion. And to do so, we have to start with a free body diagram. So it's moving, and there are forces on it caused by springs and by dashpots and by the external forces.

So right now, we're going to be free body diagram but only the components through the spring. Springs. So tell me what happens. What does that spring-- how does this spring appear on that free body diagram.

AUDIENCE: [? I have an idea. ?] [INAUDIBLE] it's air force is 11 [INAUDIBLE].

J. KIM VANDIVER: OK, force of.

AUDIENCE: Force of the spring on the--

J. KIM VANDIVER: Which direction?

AUDIENCE: Left. [LAUGHS] I don't understand how you know which--

J. KIM VANDIVER: OK, well that's kind of the point of the exercise. And I expect many of you to have some confusion about what direction to go because you don't have a standard method by which you approach this.

AUDIENCE: So both springs and cause of force goes in the direction of the displacement.

J. KIM VANDIVER: OK, so what's the displacement, though? This system has two possible displacements.

AUDIENCE: [INAUDIBLE]. We're assuming m_1 is going in the positive x_1 [INAUDIBLE].

J. KIM VANDIVER: OK. So if you said if you move it a positive x_1 , then you will get a spring force. Spring one does what?

AUDIENCE: It would oppose the [? spring. ?]

J. KIM VANDIVER: Opposes, and what's its value?

AUDIENCE: k_1x_1 .

J. KIM VANDIVER: k_1x_1 . So that's what a motion x_1 causes-- that's a result with spring one. What's the result of spring two if you have motion x_1 ?

AUDIENCE: Same. There's going to be compression force that's also opposing displacement.

J. KIM VANDIVER: Right, because it's trying to squeeze that spring down, and it's pushing back, right? But you're also going to get a spring k_2x_1 , right? Now are there any other forces that result from the motion of that body in the springs? Just spring forces now.

AUDIENCE: From that body by itself?

J. KIM VANDIVER: Yeah, that body by itself. No. But is that all the spring forces? No. So now what if you let body two move? And now I'm going to give you a little rubric, a little way to go about this.

We start by assigning our coordinates. Then you specify positive x_1 , \dot{x}_1 , x_2 , \dot{x}_2 , one at a time. And from that, you deduce the direction of the forces, of the resulting forces. OK.

So if what we've done there is consistent with it, a positive value of x_1 , spring one pulls back $k_1 x_1$, spring two pushes back $k_1 x_1$, then that's the end of spring forces due to x_1 .

So now just let's say, OK, let x_1 be 0 for a moment. And now let there be a positive x_2 . Does that cause any spring force on mass one? What do you think?

AUDIENCE: We're going to pull mass one [INAUDIBLE].

J. KIM VANDIVER: That one-- a positive x_2 puts tension in the spring and it pulls it in that direction. So now you're going to get a $k_2 x_2$. Any other spring forces caused by motion of two? It's the only connecting spring. So those are the spring forces on mass one.

Now let's move on to dashpot forces. And you do exactly the same thing. Assume a positive value of \dot{x}_1 . What does it cause in dashpot forces? What's the first dashpot do when you pull positive \dot{x}_1 in that direction? Resist or not?

AUDIENCE: Resist.

J. KIM VANDIVER: Dashpots resist. So it's going to be in the minus direction and a value-- how big?

AUDIENCE: c_1 and [INAUDIBLE].

J. KIM VANDIVER: So now you get dashpot forces, $c_1 \dot{x}_1$. And how about the c_2 dashpot?

AUDIENCE: Same thing.

J. KIM VANDIVER: Same thing. $c_2 \dot{x}_1$. OK. And that's the only dashpot forces caused by a velocity of mass one. So now let that be 0 and cause velocity at the other places and find out if anything happens.

So now let \dot{x}_2 be positive. What do we put on free body diagram?

AUDIENCE: [INAUDIBLE].

J. KIM VANDIVER: Going which direction? Positive, and what value?

AUDIENCE: $c_2 x_2$ dot.

J. KIM VANDIVER: All right. And is our free body diagram complete?

AUDIENCE: mg .

J. KIM VANDIVER: Oh, yeah. And?

AUDIENCE: [INAUDIBLE].

J. KIM VANDIVER: Yeah, and?

AUDIENCE: The forces, the external forces.

J. KIM VANDIVER: Now it's complete. All the forces in the system, it's constrained in this direction. So we just know N equals mg . There's no motion allowed, so we get no equation of motion in that direction. We're going to get one equation of motion for this mass and one more equation of motion for the second. The number of equation of motions equal the number of independent coordinates. And we found two independent coordinates. We get two equations of motion. So take this and write down the equation of motion for mass one. Sort it out. Yeah, Betsy?

AUDIENCE: So I'm really confused by why c_2 [INAUDIBLE] in that direction, because so [INAUDIBLE]. They oppose like the springs that [? felt ?] [? the closest ?] [INAUDIBLE].

J. KIM VANDIVER: OK. So let's see if we can do something here. So, this is second mass. This is the first mass. Actually, here's the wall and the first mass. So x_1 is in that direction. If I pull that way, the spring pulls back on the mass. That's obvious to you, right?

And on the other side, if this is mass one now and you have a second spring over here, a second mass over here and a spring in between them, if you put a positive motion of mass one, this spring pushes back. So that counts for the minus direction

of k_2x_1 . Then if the second mass moves in that direction-- here, you move it. You're the second mass. Is there tension in this? Am I pulling back to resist you are or not? What is this spring doing to my hand? It's pulling that way on it, k_2x_2 .

AUDIENCE: OK.

J. KIM VANDIVER: OK. So that's our free body diagram. Write right out an equation of motion for that system.

I'll give you a little reminder. That's how you ought to begin, right? And if you're done, do the same thing for the second mass while the others are working on the first one. Draw a free body diagram of mass two and write down the equation of motion.

AUDIENCE: Shouldn't that be x_2 dot?

J. KIM VANDIVER: Which one?

AUDIENCE: The bottom left, c_2x_2 dot.

J. KIM VANDIVER: All right, so let's build this. I'm just going to talk through it rather than have you help me build this. The sum of the external forces is what all of the external forces and only the external forces should show up on that diagram. The mass times the accelerations, the sum of the external forces-- so every arrow on that diagram ought to appear over here in the direction of whichever this equation is. So this is the equation that has to do with the motion of mass one.

And I look over here, and I see, OK, I'm just going to go top to bottom, minus k_1x_1 minus k_2x_1 -- and the minuses are coming from the directions of the arrows-- minus c_1x_1 dot. And then I have plus k_2x_2 plus c_2x_2 dot plus F_1 .

AUDIENCE: And a minus c_1x_1 .

[INTERPOSING VOICES]

J. KIM VANDIVER: Did I miss one? Oh, I missed the c -- I missed this guy, right? Minus c_2x_1 dot. OK.

So the arrows tell me the signs. And all the forces are there. They just all add up over here on this side. And I've gone ahead and drawn what I think is the right thing for mass two. But you do the same system. You go to mass two and you say, OK, positive deflection of mass one. What is the spring force that results? Well, it pushes on it, positive velocity of mass one. This dashpot pushes on it.

Positive deflection of mass two-- two springs push back. Positive velocity of mass two-- one dashpot resists. And you have an external force. And you could write this one down. You just add up all those forces. And now you have a second equation, $m_2 \ddot{x}_2$ equals a you add it up. OK.

So I want to do one other thing. I'm going to ask you a question, get you to raise your hand. I want everybody to try to raise their hands, OK? Assume that I had written out that second equation. Actually, let me take one more step. Normally what we would do is to rewrite this in sort of a standard form is to move all the motion variables to one side and all the actual exciting forces to the other. And so we'd say, $m_1 \ddot{x}_1$ plus, and then normally you put in the \dot{x} terms next. So we have a c_1 plus $c_2 \dot{x}_1$ minus $c_2 \dot{x}_2$ plus-- now I get my k terms-- k_1 plus $k_2 x_1$ minus $k_2 x_2$ equals F_1 . And that's kind of standard form now.

I get a second equation from the second one that would look like that. Now once you get it in this sort of standard form, can you remember how, if you've even ever had linear algebra, write this in matrix form?

AUDIENCE: It would be [INAUDIBLE].

J. KIM VANDIVER: So how many of you feel comfortable that you could just sit down and write this out in matrix form? Go on. And how many of you think it might be a bit of a challenge? OK. How many of you would like me to show you what it looks like in matrix form and give you the answer? So most of you. OK, I'll take the time to do that. Actually, did I write it out already? Ooh, look at that. Bing! Ding, ding, ding!

All you need to know in this course about vectors and matrices is how to multiply a vector times a matrix. So here's an acceleration vector. Here's the mass matrix. And

you take these two elements, my two fingers here, and multiplying by those-- you go like this to that. So x_1 double dot times m_1 double dot gives you this term, right? And you get no $m_{2 \times 2}$ double dot, because there's a 0 in the matrix in that second position. So you get no x_2 double dot term. And that's correct.

Same thing with the-- here's the x_2, x_1 . Here's this damping matrix. This is the stiffness matrix. And so if you just did these multiplies, you'd get back two equations, the ones we just derived.

So you need to know how to do this kind of thing because we write that h equals I omega. And our omega has three components, omega x , omega y , and omega z . And you need to be able to do that manipulation.

All right, we've got a little bit of time left. I wanted to do one little exercise that had to do more with this sort of thing, to kind of reinforce a bit what we did yesterday. And that would be-- so any final questions about this? Yeah?

AUDIENCE: So setting that up looks OK. But solving it.

J. KIM VANDIVER: Oh, solving it? I used to teach-- I taught for about 20 years a course called Mechanical Vibration. I just don't have time to teach it anymore. And that's what you do in there. Oh, in 803 you do a little bit of it.

So this system has two degrees of freedom. It has two natural frequencies, two mode shapes to go with it. So the system vibrates. And it's not real hard to solve, but you would assume for initially, if you want to know natural frequencies, you set the forces to 0. You set the damping to 0 initially. And you just assume a solution of the form that x_1 of t and x_2 of t equals some unknown constants, cosine omega t . Just plug it in. You just plug that in. You'll end up with an algebraic equation. And the algebraic equation actually has eigenvalues.

AUDIENCE: [INAUDIBLE].

J. KIM VANDIVER: And you find the two eigenvalues, and they're the two natural frequencies. All right. I want to do something with angular stuff. So here's my system that we were playing

with at the beginning of class. And in this form, it's nice and balanced.

So this is just to reinforce a couple things that I was just beginning to teach you yesterday. This is a rigid body. It's rotating. I attach to it a rotating frame. At a in the x direction is y and z. So y's into the board. And that frame rotates with my system.

And the rotation rate of this system is ω , and it's k in the rotating system, which happens to line up with big K with the stationary system. But this is my ω vector. Here is $0, 0, \omega$. This is the z component of the rotation.

Now, let's first start with this. What is P_1 , [\hat{c} an 0 , \hat{c}] the linear momentum of that mass in the system? And you've done this probably a lot of times. So what direction's it in? Momentum is mv , right? So what's the velocity of the mass?

AUDIENCE: It's rotating.

J. KIM VANDIVER: It's caused by rotation and only rotation.

AUDIENCE: So it could be the [\hat{c} direction. [INAUDIBLE].

J. KIM VANDIVER: No, P , just P . Linear momentum.

AUDIENCE: That P itself-- linear because it's [INAUDIBLE].

J. KIM VANDIVER: And it's got to [\hat{c} an 0 , \hat{c}] right? I mean, reference to the 0 frame. But it can use unit vectors in the A frame. So what's v_1 ?

AUDIENCE: \hat{j} .

J. KIM VANDIVER: \hat{j} , and how big?

AUDIENCE: ω .

J. KIM VANDIVER: I hear an ω times what? So this is system-- I'm going to call this position, this is x_1z_1 . This coordinate is x_1z_1 . So if I give you that information, what's the velocity of that point?

AUDIENCE: ωx .

AUDIENCE: x.

J. KIM VANDIVER: Omega x_1 --

AUDIENCE: Oh, j hat.

J. KIM VANDIVER: j hat. It's into the board, the way it's spinning into the board. Yeah, ought to be in the j direction. Yeah, it ought to be an r omega, and it has an m associated with it.

What's P2?

AUDIENCE: In the [INAUDIBLE]?

AUDIENCE: Negative [INAUDIBLE].

J. KIM VANDIVER: It's coming out of the board at you.

AUDIENCE: Right. Negative m_2 omega x_j .

J. KIM VANDIVER: Right? OK. And those little j's-- the j hats are in the rotating coordinate system where I want them. So what's H1? It's r_1 with respect to what?

So you can't do angular momentum without picking a--? You've got to pick the point. So this is why we choose the coordinates on the reference frame that we're going to use to give us some information that we want. I want to know the torques about this point. So I'm going to put my A here. I could have put it any place, as long as the axis of rotation passed through it.

So this one is r_1 with respect to A cross P1. So what is r_1 in this system?

AUDIENCE: x_1 .

J. KIM VANDIVER: $x_1 i$ --

AUDIENCE: Plus z--

J. KIM VANDIVER: k cross with m_1 omega $x_1 j$. All right? Let's work that one out. So we have an i

times a j. And so I get an $m_1 \times 1$ squared omega k. And then I do this term times that. I get a k cross j minus i. And I get $m_1 \times 1 \times 1$ omega i hat. All right.

Technically this is my little h, this is little h2. And if I let m_1 equal m_2 here, my total h for this system--

AUDIENCE: Isn't that just--

J. KIM VANDIVER: Pardon?

AUDIENCE: This is [INAUDIBLE].

AUDIENCE: Yeah.

AUDIENCE: The second minus--

AUDIENCE: The second minus is still h1.

J. KIM VANDIVER: Oh, whoops, how'd I do that? Yeah, I hadn't moved on to h2 yet. Sorry about that. This is still h1. OK. And it's that.

So what's h2? How's it differ? Just kind of look here. It only differs in one respect.

AUDIENCE: Negative sign?

J. KIM VANDIVER: Yeah, you get a minus-- a negative x here when you go to carry out the multiplication. So now the i cross j term gives you a--

AUDIENCE: But isn't the P2 also have a negative under it.

J. KIM VANDIVER: Yeah, you're absolutely right. So you get-- you had to put a minus x here. And you have to multiply by this one. So the i, j term, you have a minus times a minus. i times j is a positive k. You end up with $m_2 \times 1$ squared omega k, the same direction of each component. Then what happens here?

AUDIENCE: Comes from a minus [INAUDIBLE].

J. KIM VANDIVER: OK. And so the second term, the i term, is opposite direction. And if m_1 equals m_2

and you added these two together, what happened to those last two terms? They cancel, right?

[LAUGHTER]

OK, so if this is true, the sum of these two, H , with respect to A , is just $2m \times 1$ squared ωk . All right.

So if you took-- in the first system, if these two weren't equal and they didn't cancel out, if you go through here and take dh/dt , you take the time derivative of these terms, then you get an $\omega \dot{k}$. And over here you get an $\omega \dot{\theta}$ term. And then you get a $d^2\theta/dt^2$ term, all that.

But you will end up with values of dh/dt that are not in the k direction. You agree? For sure. And those are associated with the real torques in the system that happen.

When this is true, that difficult term out here cancels out. And you're left only with the k term. And when you take this time derivative, you only get dh/dt . You get a $\theta \dot{\omega}$ or $\omega \dot{\theta}$.

And that says the torque is in what direction when you get just this for h ? What is the direction of dh/dt ? It's still in k . So the torque is aligned with the spin. h is aligned with the spin. They're all aligned. And the system is actually beautifully balanced. You don't feel any torques around where you're holding it down here at A . But if I took one of these off, kind of back to that one arm system, it's terribly unbalanced.

So the moral of the story here is if I don't want to have these unwanted torques, what can you say about the desired mass distribution in the system?

AUDIENCE: [INAUDIBLE].

J. KIM VANDIVER: I hear a vote for symmetry. And that's the general rule. If your masses are distributed symmetrically about the axis of spin, no off axis torques.

AUDIENCE: And it's [INAUDIBLE] mass [INAUDIBLE].

J. KIM VANDIVER: Yeah, it's symmetric. Symmetry means they better be the same size too, not just in the same place. If one's twice as big as the other, no go. If I put a second mass on one of those arms, the system is back to being unbalanced.

So any time this H vector, the total H of the system, is not lined up with the spin, the system has an asymmetry in it. And the system will require additional torques just to hold it in place when it's spinning. So when you have these additional torques that come from being a symmetric, the system is said to be dynamically imbalanced.

So if you pick up a stone, a big stone, in your car wheel or block hunk of mud or ice that's frozen on the rim, you're going down the road, what's it feel like? You ever had an unbalanced tire on your car? Boy, where have you guys lived?

An unbalanced tire on a car, you're going down the road. You know? Right? That's what this is all about. You have a case of imbalance. How are we doing on time? A couple more minutes.

So imbalance comes from having angular momentum that's not pointed in same direction as-- angular momentum not pointed in the same direction as the angle of spin is evidence of unbalance. And you can calculate how bad the unbalance is by doing dh/dt and actually finding out how much torque is being applied to the system that ordinarily the bearings wouldn't have to resist. But this thing has overturning torques that are trying to make it wobble back and forth on you.

So one last exercise, which is actually a quite important point. If I moved A up here right on the line between these two things-- so this is A . And now this is x , and this is z . The coordinates of this point become what? x_1 and--

AUDIENCE: 0.

J. KIM VANDIVER: 0. And all these equations still apply. So if I just move my coordinate system so the x 's are the same, z 's go to 0. What do you end up with for terms? What happens to this thing? That goes to 0. It goes away. And you only get this term, and so by simply moving A to here, my angular momentum vector now lines up with the spin.

Kind of weird. Does the imbalance still exist? Will your car still be doing this down the road just because you chose to look at it from a different point of view? Yeah. I mean, the car's still unhappy. So there's kind of inconsistency here, sort of. What's the problem? Or maybe not a problem, but you choose to put A when you're computing angular momentum. You can choose where you put this point to give you the information you're after.

And in this case, if I were designing this system and I wanted to know the bending moment in these bars sticking down here, I put A here because it'll give you the torques with respect to this point. These torques down here still exist. If you put your point at which you compute angular momentum up here, you won't be able to find them. You can't compute it because you've reduced the moment arm to 0, this moment arm, and you just won't get that torque. OK.

All right, I've run out of time. But there'll be more on this subject.