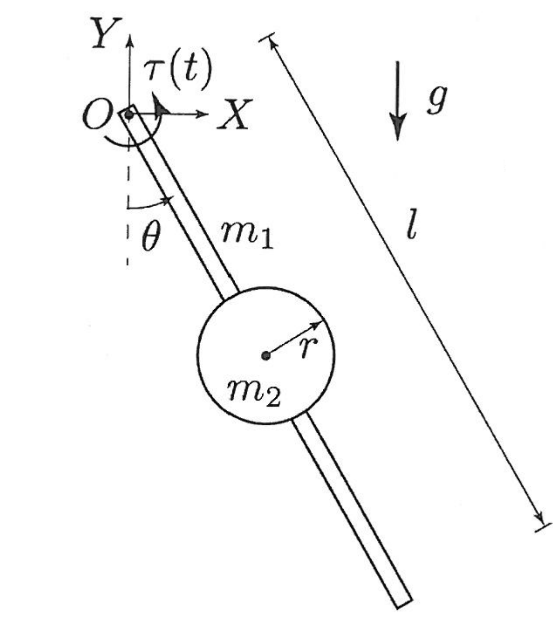
2.003SC Engineering Dynamics
Quiz 2

Problem 1 (25 pts)

A cuckoo clock pendulum consists of two pieces glued together:

- a slender rod of mass m_1 and length l, and
- a circular disk of mass m_2 and radius r, centered at the slender rod's midpoint.

The pendulum is attached at one end to a fixed pivot, O, as shown below, where a time-varying torque, $\tau(t)\hat{K}$, is applied as well. Note that gravity acts.



a) *(8pts)* Find the expression for the pendulum's mass *moment of inertia* I_{zz} *about* O.

b) *(4pts)* Find an expression for the pendulum's *angular momentum about* O.

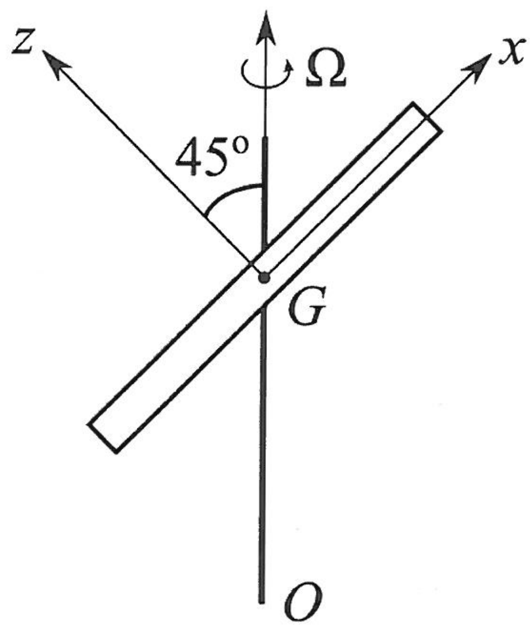
c) *(5pts)* Draw a *free body diagram* for the system.

d) *(8pts)* Find the equation(s) of motion of the pendulum *by the direct method.*

Problem 2 (25 pts)

A thin disk rotates about an axis which passes through the center of mass of the disk. The disk is inclined at 45° angle with respect to the axis of rotation as shown in the figure. G_{xyz} are body fixed principal axes and the inertia matrix for the disk is given as

$$[I_G] = \begin{bmatrix} \frac{mR^2}{4} & 0 & 0 \\ 0 & \frac{mR^2}{4} & 0 \\ 0 & 0 & \frac{mR^2}{2} \end{bmatrix} \text{ in the } G_{xyz} \text{ body fixed coordinates.}$$



a) *(9pts) Find the angular momentum of the system with respect to the* G, the center of mas of the disk. Express your answer in terms of the three vector components: $\vec{H} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$.

b) *(10pts)* Find the *torque, which must be applied at G to cause this disk* to rotate as shown in the figure. Do not assume that the rotation rate Ω is constant.

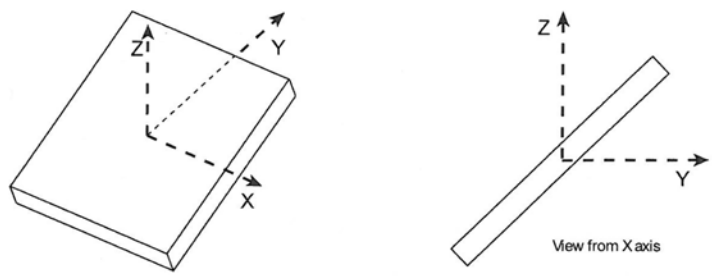
c) *(3pts)* Is this rotor *statically balanced*?

d) *(3pts)* Is this rotor *dynamically balanced*?

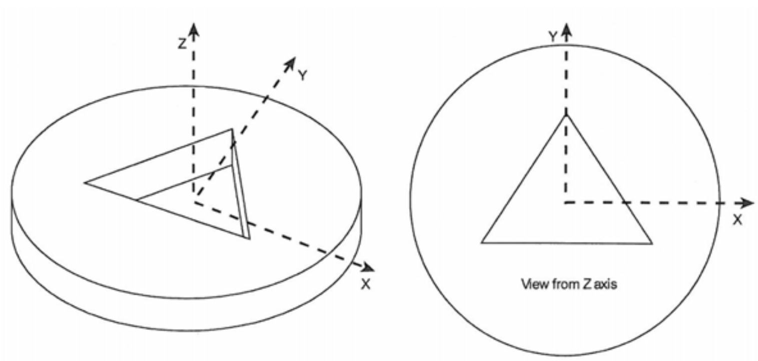
Problem 3 (9 pts)

For each of the following uniform density objects, determine whether the set of axes depicted are a set of *principal axes.* Note that two views are provided for each object.

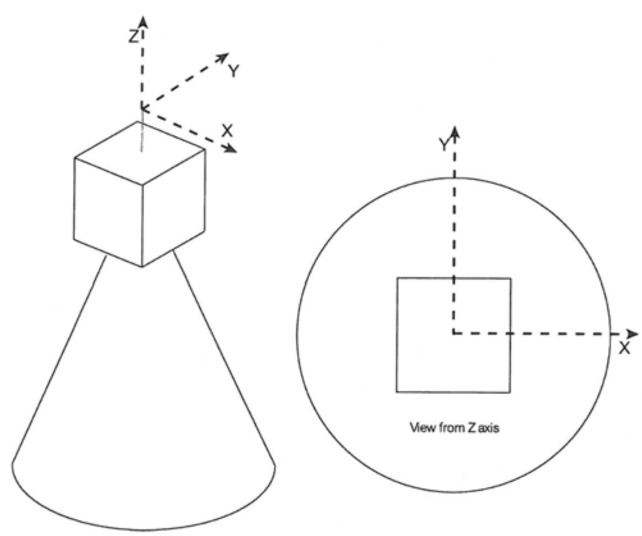
a) *(3 pts)* The axis shown are principal axes: TRUE or FALSE



b) *(3 pts)* In this object the triangular cutout is an equilateral triangle, centered in the disk.
The axis shown are principal axis: TRUE or FALSE



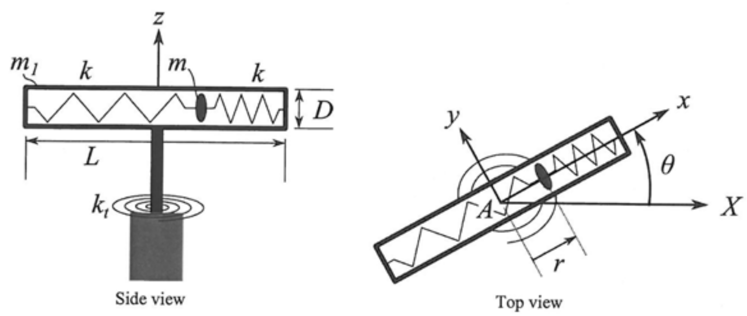
c) The axis shown are principal axes: TRUE or FALSE



Problem 4

A massless vertical rod is welded to a horizontal slender tube of length L, diameter D and mass m_l. the vertical rod is supported in a frictionless bearing. Attached to the vertical rod is a torsional spring with spring constant k_t with units of Nm/rad. A mass m is attached to both ends of the tube, with two springs, each of spring constants is k, and unstretched length is $L/2$. The mass slides frictionlessly in the tube. The mass may be treated as a point mass. The inertia matrix for the tube expressed in its body fixed axes A_{xyz} is given approximately by:

$$[I] = \begin{bmatrix} \frac{1}{4}m_1 D^2 & 0 & 0 \\ 0 & \frac{1}{12}m_1 L^2 & 0 \\ 0 & 0 & \frac{1}{12}m_1 L^2 \end{bmatrix}$$



(θ, r) defined on the top view of the figure form a set of complete and generalized coordinates, which describe this two degree of freedom system. You should not assume that (θ, r) or their first two time derivatives are zero.

a) *(15pts)* Calculate the *kinetic energy T, potential energy V, and generalized forces* Q_r and Q_θ for this system.

b) *(10pts)* Derive the *equations of motion* for the system *using Lagrange* method and using the generalized coordinates (θ, r).

Problem 5

Two blocks of mass m_1 and m_2 are frictionlessly constrained to vertical motion. The first block is connected to the ground via a spring and a dashpot with constants k_1 and b_1 as shown. the second block is connected to the first one via a spring and a dashpot with constants k_2 and b_2. A force $F(t)$ is applied to the second mass as shown of the figure.

The vertical position of the first block is denoted by x_1 while the position of the second one is denoted by x_2. (x_1, x_2) form a set of complete and independent generalized coordinates to describe this two degrees of freedom system.

a) *(8pts)* Calculate the generalized force Q_1 associated with the generalized displacement x_1.

b) *(8pts)* Calculate the generalized force Q_2 associated with the generalized displacement x_2.