# 2.003 Modeling, Dynamics, and Control I
# MIT
# Department of Mechanical Engineering
# Spring 2005
# Prelab 6

Week of April 4, 2005

## Purpose of Lab

The primary purpose of this lab is to understand the response of a flywheel/damper system (the same one you used earlier in the semester to look at first order response) when driven by a motor. This will pave the way to closed-loop control with the same hardware in a subsequent lab. A secondary purpose is to understand the difference between driving a rotational inertia directly and driving it through a gear reduction.

## Hardware

In this lab we will be using the hardware shown in Figures 1 and 2 below - an elaboration of the hardware you used earlier in the semester. Two features have been added since you encountered this hardware a few weeks ago. Figure 1 shows a pair of *spur gears*. As a result the flywheel can either be driven directly from the *flywheel shaft* or through a gear train by driving the *gear shaft*. The small and large gears have $n_1 = 44$ teeth and $n_2 = 180$ teeth respectively, and thus have a gear ratio of 4.09:1. Figure 2 shows the addition of a permanent magnet DC motor which is coupled (through a bellows coupling) to the gear shaft.

## Questions

For the sake of discussion, let's initially assume that the motor acts as an ideal torque source, and does not add any damping, friction, or inertia to the system.

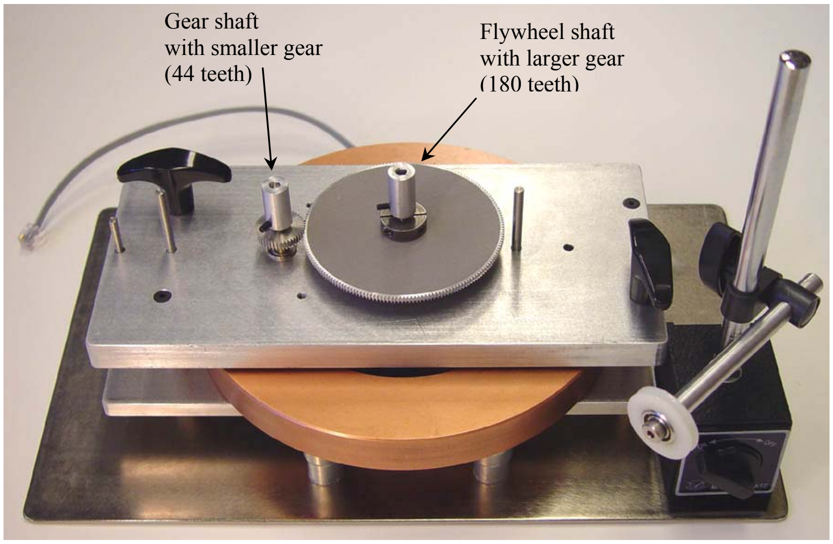


Figure 1: Flywheel apparatus with gear train.

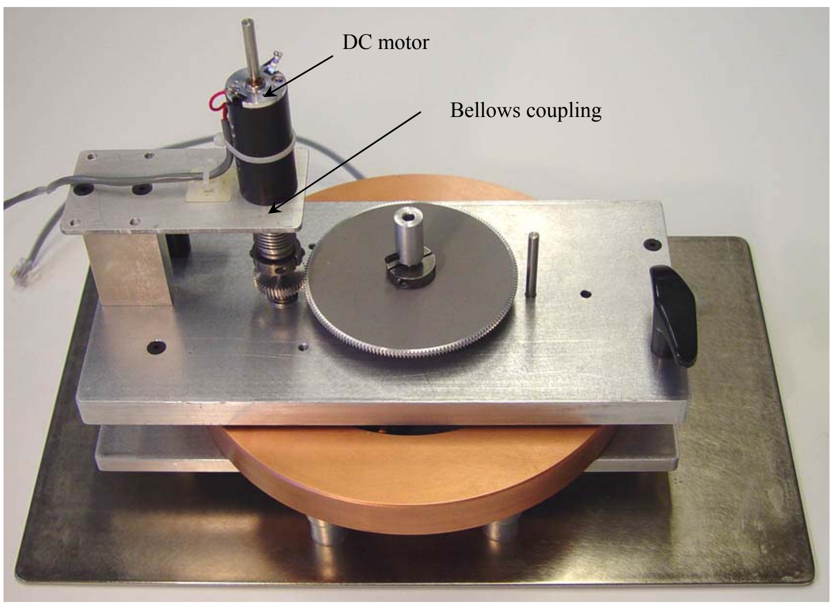


Figure 2: Flywheel apparatus with gear train and DC motor.

1. Step response, direct and through gear reduction

   (a) Suppose that a torque source is coupled directly to the flywheel shaft in our setup and that the rotational inertia has a value J and the eddy-current damping has a value B. Sketch the angular velocity of the flywheel as a function of time in response the step change in torque shown in Figure 3. What is the time constant of the response? What is the steady state velocity after the transient? What is the maximum acceleration of the flywheel and when does it occur? Show the steps in your modeling process.

   (b) Now suppose that the same torque source is coupled to the gear shaft. Develop a model which includes the gear reduction. On the same plot that you created for (a), plot the

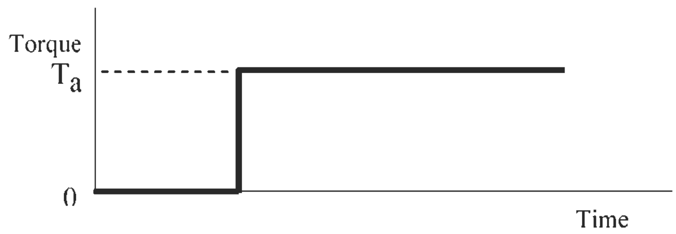


Figure 3: Torque as a function of time $T_a$

response of the flywheel angular velocity to the same step change in torque as applied in (a). Again, what is the time constant? What is the steady state velocity? What is the maximum acceleration of the flywheel and when does it occur?

(c) Discuss the relationship of your answers to (a) and to (b). Discuss the relationship between the time constants in the two cases. Discuss the relationship of the steady state velocities. How can they be different if the same torque source is used in both cases? Think physically and verify via analyses with your model.

2. Now we want to look at the details of the motor and its drive amplifier. The system is configured as shown in Figure 4.

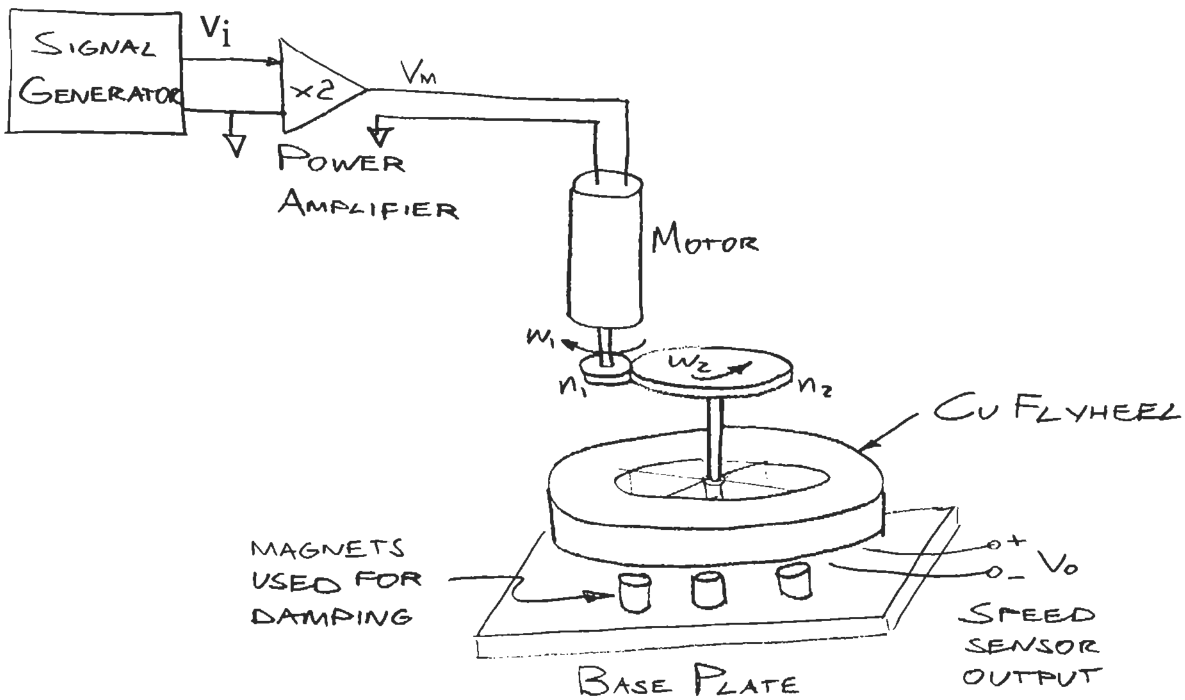


Figure 4: System configuration.

The system also uses an encoder and associated electronics as a speed sensor with a gain of 0.31 [V/rad/sec]. Note also that the amplifier has a gain of 2 such that $V_m = 2V_i$.

(a) A manufacturer's data sheet for the motor is attached. The motor model number is 118745. From this data sheet, what are the values of the motor:

| | |
|---|---|
| Torque constant | $K$ [Nm/A] |
| Coil Resistance | $R$ [$\Omega$] |
| Inertia | $J_m$ [kg m$^2$] |

Be sure to report these numbers in MKS units as indicated.

Figure 4 shows magnets on the baseplate which can be used for adding variable damping, which we will model as $B$ [Nm sec/rad]. The flywheel inertia has the numerical value $J = 0.03$ [kg m$^2$].

(b) For now, assume $B = 0$, that is the magnets are removed, and thus the only source of damping comes from the motor. Develop a mathematical model for the system. This model should include the motor inertia, but assume the gear inertias are zero. Calculate the transfer function $V_o(s)/V_i(s)$. Plot the transfer function pole on the complex plane. Also calculate and plot the response $V_o(t)$ to a unit step $V_i(t) = 1$ V $\mathrm{u_s(t)}$.

(c) Now, let $B = 0.0375$ [Nm sec/rad] which models the magnets in place, and repeat part (b). How do the time constant and step response vary?

Datasheet removed for copyright reasons.
Maxon RE25 DC Motor.