MASSACHUSETTS INSTITUTE OF TECHNOLOGY DEPARTMENT OF MECHANICAL ENGINEERING CAMBRIDGE, MASSACHUSETTS 02139

Due Week of May 3-6, 2004

2.002 MECHANICS AND MATERIALS II LABORATORY TERM PROJECTS

OVERVIEW

The term project will serve as a capstone to the lab work performed in this class. Each lab section is to be divided into groups of between three and five students. Each group is expected to choose one of the projects listed subsequently (or to propose and receive approval for an alternative project); however, no two groups in the same lab section should choose the same project.

The suggested projects fall into a small number of categories, which can be broadly described as

- quantitative design, analysis, construction, and mechanical testing of a simple structure or component
- quantitative experimental exploration of fundamental aspects of mechanical behavior of materials
- quantitative experimental comparisons of mechanical behavior: competitive commercial products, competitive materials, competitive designs

In the following sections, a selection of suggested activities within each category are briefly outlined, including references to related background information. Remember, these are meant to be *suggestions* only, and if you have interests in pursuing other topics, you are encouraged to do so!

There are three principal expectations concerning your lab project:

- 1. you will perform reasonably precise mechanical tests on components, structures, and/or material specimens
- 2. you will perform a suitable mechanics-based quantitative analysis of your tests and results; the detail and level of sophistication of the analyses range from free body diagrams to simple stress and deformation analysis (e.g., beam theory models) to detailed analyses based on elasticity, plasticity, etc.
- 3. you will prepare a written report of your project, as well as a team-based presentation to your lab group

A. Component Design, Analysis, Construction, and Testing

1. Circular ring load cells

One way to construct load cells of modestly high resolution is to 'amplify' the load-induced strain by means of bending deformation in a thin structural member. This principle was embodied in the strain-gaged cantilever leaf spring of Lab 1. An in-line load cell utilizing bending (also called a "proving ring") can be constructed from a short axial section of a thin-walled circular tube, as shown schematically below: The cross-section of the cell is a ring of mean radius R and wall



Figure 1: Schematic of a ring-shaped load cell. Side view.

thickness t; the axial extent of the ring is B, as shown. Viewing the ring as a "clock-face", equal and opposite tensile loads P are applied at 12 and 6 o'clock, respectively. Elastic bending behavior of the ring walls causes a state of combined tension and bending at the 3 and 9 o'clock locations so that the inner and outer walls experience different circumferential strains, and the thin-wall ring-bending provides good amplification of surface strain compared to (small) average strain.

In practice, load can be transmitted through the ring by making B sufficiently large so that, for example, a bolt or other connector passes through a radially-directed hole at 12 and 6 o'clock; however, B should not become too large compared to R, or else the structure starts to behave much more like an elastic "shell" than an elastic "ring."

Resources: Simple but accurate analysis of the stiffness and strain and stress distributions in a thin ring of this sort, under transverse loading, can be carried out by making use of so-called "energy-methods" of structural analysis (sometimes referred to as Castigliano's theorem). Chapter 8.6 of the CDL text presents the beginnings of a structural analysis of curved planar beams using Castigliano's theorem (see also CDL problem 8.23); more complete information on carrying out the analysis for diametrically-opposed loading of the complete ring can be found in the Dover text by J. Den Hartog on *Strength of Materials*. Professor Parks is also available as a resource for setting up the complete analysis (once set up, the analysis involves only elementary integration and solving a system of 3 linear equations in 3 unknowns).

Your project would comprise (a) study of the structural mechanics solution; (b) calculation of overall stiffnesses, and relations between load P, ring geometry and material properties, and local strains; (c) construction and instrumentation of a ring load cell, and comparison of your experimental and theoretical results.

2. Energy absorbing structures: tube crushing

It is often desirable for ductile structures to be designed to dissipate large amounts of [kinetic] energy through plastic deformation, without generating excessively high loads or accelerations. Common applications include "energy-absorbing" design in ground vehicles.

One simple design concept for dissipating plastic deformation under imposed compressive loading can also be based on the thin ring geometry of the previous figure, but this time the intent is to reverse the sign of the opposing loads, and, moreover, to imposed large diametral shortening on the 6/12 o'clock axis by forcing planes together sufficiently far to cause fully plastic bending behavior in the wall of the ring (recall the sheet-bending experiments in Lab 3) at so-called "plastic hinges" located initially at 3, 6, 9, and 12 o'clock. The sign of the fully plastic bending moment and curvature alternates as one proceeds circumferentially around the ring.



Figure 2: Schematic of lateral crushing of a short tube segment. Side view.

Resources: The theory of plastic limit analysis provides a relatively simple algebraic means for estimating the initial maximum compressive support load of the ring under compression, depending on the ring geometry and tensile flow strength, σ_y . The beginnings of a discussion of plastic hinges and limit analysis in beams can be found in CDL Chapter 8.7; further details are available in texts on plastic structural mechanics. In particular, see Chapter 7.4 of *Engineering Plasticity*, by W. Johnson and P. B. Mellor, Van Nostrand, London, 1973). Professor Parks can also serve as a resource for setting up an initial limit analysis of this problem.

Of course, once the ring starts to deform plastically, its geometry changes substantially; ultimately the ring becomes highly elongated and flat. It is difficult to follow the changing geometry with an analytical model, but modern numerical methods based on the finite element method provide capabilities here. If this topic is chosen by a lab group, we will secure the services of a graduate student or post-doctoral researcher to work with the group in conducting a suitable finite element simulation of large crushing behavior.

Your project would comprise (a) study of the plastic limit analysis solution; (b) calculation of peak support loads in terms of ring geometry and material properties; (c) construction and testing of ring-crush structures, and (d) comparison of your experimental and theoretical (limit analysis and [optionally] finite element) results.

B. Fundamentals of Mechanical Behavior of Materials

1. Symmetry of Tension & Compression True Stress vs. True Strain Curves

As was noted in the lecture, it is commonly accepted that the true stress vs. true (logarithmic) strain curves of many materials (especially metals) essentially coincide. In this project, the objective is to perform both tension <u>and</u> compression testing of various materials in order to determine quantitatively how closely the tension and compression true stress-true strain curves coincide.

In particular, a lower-strength metal exhibiting high strain hardening, a higher-strength metal exhibiting low strain hardening, and the engineering polymer polycarbonate (PC) should all be tested in both tension and compression. For example, a steel such as 1045 could be heat-treated to both a hard Q&T condition, as well as a softer one.

2. Uniaxial large-strain plastic behavior of polycarbonate

Earlier we tested polycarbonate in an "as-received" condition, and noted elastic behavior, an initial plastic yielding, and finally, pronounced strain hardening as the polymer chains start to preferentially elongate in the direction of maximum principal stretch and strain. A careful study of the true stress versus true strain in these materials reveals additional complexities.

For example, there is a noticeable effect of the [magnitude of] true strain rate on yielding, with faster deformation corresponding to higher levels of flow stress. The local peak in stress associated with initial yielding is generally noticeably larger in compression than in tension (although the difference is not large, typically no more than 10% or so). The magnitude of the strain at which the rapid upturn in true stress occurs (due to the pronounced polymer chain alignment) is markedly different, with the rapid upturn in tension occurring at a smaller strain than in compression, and a steeper hardening slope in tension than compression. The magnitude of stress at the initial peak (initial yielding) depends strongly on prior thermal history of the sample. Samples that have been held without applied stress for various times, at a temperature just below the glass transition temperature, can show markedly increased peak-values of stress, but they subsequently decay to similar values at larger imposed strain. This phenomenon is sometimes termed "ageing" of the polymer, and can tend to embrittle the material.

Finally, "plastic" deformation in glassy polymers such as polycarbonate is, in some sense, not truly "plastic" (or non-recoverable), in that the shape of a specimen deformed below glass transition temperature, T_g , can be ~fully recovered by increasing the temperature to above glass transition, in the absence of applied stress. If the heating is performed with deformation constrained, it is possible to generate force in the process. For example, a compression specimen could be squeezed to a fraction of its initial height, and then unloaded just to zero load, but with the loading platens just touching the specimen. Upon heating towards T_g , the specimen would try to recover its initial height, but in the process would contact the [resisting] platens, and thereby create compressive loads again.

These are just some of the unusual and fascinating behaviors that can be observed in plastically deformed polycarbonate. World-wide, the main efforts in modeling these and related mechanical behaviors in polymeric and elastomeric materials have taken place in the past decade or so here at MIT, by graduate students under the supervision of Professor Boyce (including Dr. Bergstrom!). There are many resources to be found in these students' theses and their published papers.

3. So how do brazed connections carry all that load?

Brazed connections are common ways to form strong mechanical bonds between metal surfaces. The essence of the process is that the braze material has a generally low melting temperature compared to the metals it is to connect. When the surfaces to be joined are place in close (but not touching) contact and heated, and then the heat (via a flame) is applied to melt the braze material, capillary forces "suck" the molten braze metal into the thin gap between the surfaces. When the joint cools, the braze metal solidifies, forming the solid bond. A common application of brazing is in the joining of plumbing and piping, although brazing occurs in a host of engineering applications including aerospace.

The metals forming the braze are often of lower strength (or hardness), compared to the metals that they are joining, and are usually themselves ductile. Yet planar brazes surfaces loaded in direct tension perpendicular to the surface can successfully support surprisingly high levels of load; in fact, such brazes typically support several times the tensile strength of the homogeneous braze metal itself! How is this possible?

A clue may reside in the much more "reasonable" magnitude of the **shear strength** of a planar braze area; in fact, shear strengths of brazes are in accord with the shear strength of the braze metal.

In this project, you would create model braze connections and load them in both tension and in shear. Simple limit load analyses could be performed which would point the way to understanding the enhanced tensile performance of the braze.

Resources: The theory of plane strain plastic limit analysis provides a relatively simple algebraic means for estimating the maximum tensile support load of a braze under compression, depending on the braze metal thickness/width aspect ratio, and tensile flow strength, σ_y of the [softer] braze metal. Notes on "block-sliding" limit analysis can be obtained from Professor Parks; further details are available in texts on plasticity. In particular, see *Engineering Plasticity*, by W. Johnson and P. B. Mellor, Van Nostrand, London, 1973).

4. Thickness Dependence of Fracture Toughness

We saw in Lab 6 that the plane strain fracture toughness, K_{Ic} , while conservative in predictions of the cracking behavior of thinner-sheet structures, did not reflect the thickness-dependence of crack-tip plasticity and toughness, $K_c(B)$. Figure 2 of the Lab 6 handout contained a graphical suggestion of how to "scale" fracture toughness with thickness in a 7075 aluminum alloy, but the supporting documentation was sparse.

In this project, it is suggested that an experimental study of thickness effects on toughness be undertaken in 7075-T6 aluminum compact tension (CT) specimens, leaving in-plane geometry as previously tested, but systematically reducing specimen thickness (only) from the 12.7 mm of the original plate. In practice, this involves milling these specimens to reduces thickness.

In order to isolate the thickness effect, testing should be done with nearly equivalent fatigue presharpening histories, to similar crack lengths.

When considering the testing itself, and interpretation of the results, the following points should be kept in mind.

- (a) Note that the "back face" of the CT specimen actually sees compressive stressing (it is really a bend specimen!), and in very thin sheet stock, out of plane local buckling may occur under compression.
- (b) Plane stress-type crack stable crack extension under initially rising loads is a feature that becomes increasingly prominent in thin sheet fracture mechanics testing. This means that, if the initial, pre-fatigued crack length is a_i , then, as load P is monotonically increased, the stable growth of the crack starts at some load P_i , and load and crack length both increase (somewhat) during the early portions of growth. Clearly, this complicates the issue of a 'critical' value of stress intensity factor for growth, but it could be addressed.
- (c) The lecture handout on fracture notes the trends that might be expected in the shape of crack tip plastic zone as thickness decreases, and suggested that this change also shows up in the relative proportions of "flat" and "slant" fracture surface areas (see also figure 8.46 in Dowling text). Quantify this trend in your experimental results as well.

Resources: Good reference information on plane stress fracture mechanics and testing is contained in the various texts by David Broek (e. g., *Elementary Engineering Fracture Mechanics*, 4th Ed., Nijhoff, 1985).

5. Hertzian contact

When a linear elastic spherical (or circular cylindrical) body is pressed into contact with another convex body, the contact process itself is highly non-linear, even though both materials are linear elastic. The reason, of course, is mostly geometrical: at negligible contact load, the area of contact degenerates to a point, and, as loading is applied, the area of contact smoothly increases, focussing most stress and deformation near the contact point. As summarized in Appendix A.8 to Ashby's text, *Materials Selection in Mechanical Design*, (Butterworth-Heinemann, Oxford, 1999), when the center of an elastic sphere of radius R and Young's modulus E and Poisson ratio ν is pressed toward a much stiffer flat surface by a distance v, the contact force F_c scales as

$$v = \left(\frac{9}{16} \, \frac{F_c^2}{RE^{\star^2}}\right)^{1/3};$$

the contact area A_c (a circular patch of radius $r_c = (A_c/\pi)^{1/2}$) scales as

$$r_c = \left(\frac{3F_cR}{4E^\star}\right)^{1/3},$$

and the contact pressure distribution varies with radius r according to

$$p(r) = p_{\max} \left(1 - (r/r_c)^2\right)^{1/2},$$

where

$$p_{\max} = \frac{3F_c}{2A_c} = \left(\frac{6F_c E^{\star^2}}{\pi^3 R^2}\right)^{1/3}$$

Here, with one of the contacting members both flat and rigid in comparison to the other, the modulus measure $E^* = E/(1-\nu^2)$ of the compliant sphere.

This and related elastic contact solutions were formulated and solved by a then-23-year old Heinrich Hertz (he of the famous cyles/second measure!) during Christmas break of 1880. So-called Hertz contact analysis is used ubiquitously in engineering design, on components ranging from seals (e.g., O-rings) to the precision kinematic couplings that have been used so successfully by Professor Alex Slocum.

In this project, the objective is to perform compression testing on a hard rubber ball (e.g., a superball) in order to quantify certain aspects of the Hertz solution. In particular, verify the nonlinear behavior of load vs. deflection and contact area vs. deflection (or load). Quantification of the behavior with respect to elastic moduli of the sphere could be verified by an independent measurement of elastic moduli, perhaps on a second specimen (or part cut therefrom). There exist so-called pressure sensitive films that are reported to change colors (permanently) in response to varying levels of imposed contact pressure; perhaps this technology could be investigated to verify the spatial variation of peak contact pressure.

Resources: The text *Contact Mechanics*, by K. L. Johnson, (Cambridge U. Press, 1985; see Chapter 4 and Appendix 3) contains an excellent derivation and summary of the Hertz contact solutions.

6. Acoustic measurements of elastic modulus

Recently visiting Cambridge University faculty member Dr. Hugh Hunt demonstrated in course 2.001 a highly accurate method for measuring elastic modulus based on acoustic signals in a vibrating elastic structures of known density and vibration mode.

The method involves using MATLAB data acquisition to process acoustic signals taken from a vibrating coupon of the sample material. In many ways, the method is similar in spirit to the vibrating beam experiments of Lab 1, but is generally much more accurate, owing to the lack of coupling of high-frequency vibration modes to specimen supports, in contrast to the very low frequency vibration modes we measured (with stopwatches!) in Lab 1. In this project, you would learn more about elastic waves/vibrations, and about using MATLAB features to automate data reduction.

Resources: The text *Mechanical Vibrations*, by J. P. den Hartog (Dover, 1985), Chapter 4, is an excellent intuitive source for vibration problems.

C. Comparative Mechanical Behavior

1. Double-walled aluminum softball bats

Background

Recreational softball and baseball is a huge market spanning ages from 6 to 66. While traditional bats are made of wood (typically, ash; maple), and wood is the only material allowed in professional baseball, there is considerable interest in the use of hollow, aluminum alloy bats. These structures can be made light-weight, and can be designed to provide high levels of "slugging capacity" to the user.

Getting a bit more technical, an important issue in bat performance is how deformation of the wall of the bat interacts with deformation of the ball itself during the brief interval of time when bat and ball are in contact. In turn, that time interval consists of a first portion of increasing contact load that increasingly deforms both ball and bat wall, followed by an interval of rebound during which both bat wall and ball tend to spring back to their pre-impact shapes. It was once thought by many that a very stiff, or 'rigid' bat design produced the best hitting response. However, ball impact on a very stiff structure leads to very high contact forces over a very short interval. In turn, very high contact forces can lead to high levels of deformation and energy dissipation within the ball itself; this dissipated energy could conceivably have been better used as kinetic energy of the ball!

More recently, some designers have come to understand that a lower-stiffness bat wall design might have significant advantages in this respect. As bat wall stiffness is systematically decreased, starting from a very high initial value, the resulting duration of bat/ball contact increases and the peak contact force decreases (all other variables being 'equal'). In turn, lower contact force dissipates less energy within the ball, and so on.

In essence, the idea is to temporarily 'trap' (or store) some of the pre-impact kinetic energy of moving bat and ball as recoverable elastic strain energy of deformation of the bat wall; the storage takes place in the first part of the impact interval. Then, during the second (or rebound) portion of impact, this stored elastic energy is smoothly transferred to kinetic energy in the ball as bat wall deformation springs back.

Now back to bat design. It is clear that the stiffness of a bat wall to impact depends on bat diameter, wall thickness, and material elasticity, in addition to other geometrical and structural details of the overall bat design. Focussing on softball bats, which generally have a constant-diameter cylindrical impact portion, wall thickness becomes a primary design variable; if thickness is large, the wall will become stiff to impacts. Conversely, lower bat-wall stiffness can be achieved by reducing wall thickness. However, if wall thickness is reduced too much, it is possible for local stress in the wall to reach yield values, resulting in permanent or plastic deformation (denting). Another possible manifestation of impact-induced plasticity is the development of local fatigue cracks in the bat wall after sufficient use.

The commercial struggle to optimize aluminum bat materials, design, and performance has led to remarkable advances in recent years. (Incidently, in view of safety issues related to bats that are "too hot", the industry has also imposed certain (maximum) performance limits on bat design, but that is another story.) Current-generation aluminum alloys used in thin-walled bats have tensile yield strengths up to 40% higher than 7075Al, the aerospace industry's "workhorse" high-strength aluminum alloy: higher yield strength means than thinner walls can be used without denting or fatigue.

Another design concept has been introduced to achieve reduced bat-wall stiffness to lateral loading, while limiting the magnitude of induced stress: replace a homogeneous bat tube of wall thickness

"t" with a multi-walled bat of intimately-nested independently-movable concentric tubes, each tube of lesser thickness; for example, 2 tubes, each of thickness t/2, could be substituted instead (plus many variations). Among the commercial products using such technology are the "Doublewall" bats of Wilson/Demarini and the "EST" bats of Worth. The critical feature distinguishing a multiwall bat from a homogeneous single-walled bat of equivalent total thickness is the ability of the multiple walls to slide relative to one another (or not) during lateral deformation. An analogy is the greatly-reduced 3-point bending stiffness of 2 sliding "leaf-springs" of thickness t/2, compared to a single leaf of thickness t.

This project would involve (quasi-static) mechanical testing of the lateral-loading stiffness of various aluminum bat designs. Mechanical stiffness might be correlated with (subjective?) measures of hitting performance. Yield strength might be inferred from hardness measurements. Comparative measures of "denting loads" might be established. Some specimen(s) might be disassembled to quantify internal structure.

Resources Interesting related information is contained in Chapter 5 of *The Physics of Baseball*, by Robert K. Adair (Harper Perennial 1990), however the details of the "spring effect" on impact are not addressed there.

2. Mechanical Design with Polymers

A manufacturer comes to you with a big project! She wants you to manufacture multi-purpose brackets to be used in furniture, as illustrated in Figure 4. For aesthetic reasons, the components have to be transparent. Polymeric materials polycarbonate (PC) and poly-methylmethacrylate (PMMA) seem to be good candidates for this application. The cost of 1/2" thick PC sheet is $42.50/ft^2$, while that of PMMA is $6.56/ft^2$. Both of these polymers are thermoplastics; *i.e.*, they soften when heated and reharden when cooled. Hence, components made from these materials can be thermo-formed in a mold of the desired shape. Your task as a designer is to:

- Devise and perform necessary tests to acquire mechanical properties of these materials for a successful design. Keep in mind that the properties of thermoplastics may vary significantly with temperature.
- Design the bracket so that it can withstand the specified loading (see Figure 4). Note that the load and length parameters are specified; other parameters such as the thickness, width, and radius of curvature will be determined by your design. Keep material availability in mind when selecting thickness.
- Select one from the two given polymer materials which best suits the application. Be sure to take into consideration factors such as strength, toughness, impact and scratch resistance, etc.
- Determine the thermoforming temperature and the rate and mechanical pressure to be applied to the softened thermoplastic sheet in order to achieve and maintain precise shape of the components. item Produce a prototype of the component within 3 weeks.
- Test the prototype bracket.

Although you don't know much about polymers (yet), the offer is too tempting to turn down! So you decide to undertake the project. Some references that might be useful in your design have been appended below.

References

- (a) Ashby, M.F. and Jones, David R.H., Engineering Materials 2.
- (b) Ashby, M.F., Materials Selection in Mechanical Design.
- (c) Courtney, T.H., Mechanical Behavior of Materials.
- (d) Budinski, K.G., Engineering Materials, Properties and Selection.
- (e) Standard Test Method for Tensile Properties of Plastics, D638, Annual book of ASTM Standards, ASTM.
- (f) Handbook of Plastic Testing Technology.

Figure 3: Schematic of transparent polymeric furniture bracket and loading.

3. Lightweight Core "Sandwich" Beam Construction

The markets for aerospace, athletic equipment (e.g., skis) and other industries have been driven towards designing structures of least weight while delivering desired levels of structural performance. While selection of appropriate materials is an important design consideration, so, too, is understanding how optimized performance can be obtained by the geometrical arrangement of different types of materials within a structure or component.

In this regard, beam (or plate) bending offers unique opportunities for constructing cross-sections of optimal shape. We know that, under bending deformation, the largest tensile and compressive strains occur on the top and bottom surfaces of the flexing beam, where, moreover, the correspondingly stressed material has the largest moment arm with which to resist the bending moment, M. Nearer to the neutral axis of the beam, the spatially-reduced bending strains and moment arms lead to rapidly diminishing contributions to bending resistance in any event, so there is little (if any) reason to put stiff material near the neutral axis!

One approach has been to form a composite beam cross-section by adhesively attaching thin surface layers of a stiff, light-weight metal (or composite) plate to the top and bottom faces of a less-stiff, and even lighter-weight "core" material, such as a foamed polymer or metallic hexagonal "honeycomb" structure. The thickness of the core can be increased with little cost in structural weight, all the while increasing the moment arm for the stiff face plates to increase structural bending stiffness.

However, the sandwich construction does not come without a few new items to consider in design, including:

- When a section of the beam is required to carry transverse shear forces, V, the relatively large area of the core, compared to that of the two thin faces, means that there will be shear stress in the core. In turn, the core material is likely to have a very low shear modulus, meaning that lateral shear deformation of the beam structure (usually neglected in traditional beam theory developed for beams whose cross-section is homogeneously constructed of isotropic elastic material) may contribute substantially to the overall stiffness of the structure.
- The shear stresses present at the adhesively joined interface between the core and the faces must not cause delamination or failure.
- If highly concentrated loads are applied directly to the thin face plates, very local deformation of the face plate and/or the core beneath the load point may lead to large local deformations and failure.
- If the compression-side face plate exhibits local delamination, it may be prone to further delamination through buckling-driven de-bonding.

In this project, you are to design, construct, and test a sandwich beam consisting of aluminum alloy face plates bonded to a foam core. Specifications for the span, 3-point loading, and load performance indices (stiffness, peak load, etc.) will be given later, as will notes on analysis of shear stress and shear deformation in sandwich beams.

Your task will be to:

- Devise and perform necessary tests to acquire mechanical properties of the core and face materials, as well as adhesive strength.
- Design the sandwich beam so that it can withstand the specified loading. Keep material availability in mind when selecting thickness.
- Produce a prototype of the component, including provisions for monitoring strain in the surface plates and the pattern of lateral deflection along the length of the beam.

- Test the prototype beam, comparing its actual behavior with predictions based on your analyses.
- Summarize your project in a report, and prepare a brief 15-minute presentation of your findings.