

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF MECHANICAL ENGINEERING  
CAMBRIDGE, MASSACHUSETTS 02139  
2.002 MECHANICS AND MATERIALS II  
HOMEWORK SOLUTION NO. 5

**Distributed:** Wednesday, April 10, 2003  
**Due:** Wednesday, April 16, 2003

**Problem 1 (30 points)**

The critical stress intensity factor,  $K_{IC}$ , is given by the inequality

$$K_{IC} \geq Q\sigma^\infty \sqrt{\pi a} \quad (1)$$

Solving for the critical crack length,  $a_{crit}$  we get

$$a \leq a_{crit} \equiv \frac{1}{\pi} \left( \frac{K_{IC}}{Q\sigma^\infty} \right)^2 \quad (2)$$

For an edge crack (whose length is less than or equal to 0.65 times the plate's width,  $w$ ), the configuration correction factor,  $Q$ , is

$$Q = \frac{1.12}{\left(1 - 0.7 \left(\frac{a}{w}\right)^{1.5}\right)^{3.25}} \quad (3)$$

We are told that the the plate is *large*, which implies that  $\frac{a}{w} \ll 1$ . It is also explicitly stated that

$$Q = 1.12 \quad (4)$$

We are also told to use a design stress equal to 0.6 times the minimum yield strength.

$$\sigma^\infty = 0.6 \sigma_{ymin} \quad (5)$$

Substituting equations 4 and 5 into equation 2 we get

$$a_{crit} = \frac{1}{\pi} \left( \frac{K_{IC}}{0.672\sigma_{ymin}} \right)^2 \quad (6)$$

We are asked to compare the critical crack sizes for four different grades of steel. The values of  $\sigma_{y_{min}}$  and  $K_{IC}$  for these steels are given in the second and third columns of table 1. The fourth column of the table contains the critical crack length as computed by equation 6.

Steel (ASTM Grade)	$\sigma_{y_{min}}$ [ksi]	$K_{IC}$ at $-60^{\circ}F$ , [ksi $\sqrt{in}$ ]	$a_{crit}$ [in]
A36	36	60	1.958
A441	50	53	0.792
A572	50	100	2.82
A514	100	60	0.254

Table 1: Material properties and Critical crack length for available steels

Based on table 1, the steel that would most likely be selected is A572. A572 would be chosen for two reasons

1. it has the largest critical crack size, of nearly 3 inches. Critical cracks could be easily spotted by the human eye during routine maintenance.
2. its  $\sigma_{y_{min}}$  is larger than all of the other steels, except A514. This means a smaller amount of steel would be needed to construct the bridge, likely making it cheaper.

If A36 cost less than half as much as A572, it might be selected instead on the basis of cost. (Note that the real world price of A36 is NOT less than half that of A572). This is an option because A36 also has a large critical cracksize; it is nearly 2 inches. A441 and especially A514 would be eliminated on the basis of small critical crack size.

While not enough information was given in this problem to make a decision based on fatigue, it should be noted that fatigue will be a key factor simply because the structured being designed is a bridge. This is due to the fact that bridges are frequently loaded and unloaded by the passage of cars over them.

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**Problem 2**

There are no detectable cracks, so I assume the ‘worst case’ scenario, which is the presence of the largest possible, undetected cracks, which is  $5mm$ . Since it is stated that the plate is large,  $\frac{a}{w} \simeq 0$  and it is safe to assume that the configuration correction factor,  $Q$ , is  $Q = 1.12$ . Assuming the plate will fail by brittle fracture, the stress at which this occurs can be found by

$$K_{Ic} = Q\sigma_{\infty}\sqrt{\pi a} \quad (1)$$

where  $K_{Ic}$  is the given fracture toughness,  $\sigma_{\infty}$  is the far-field, or applied stress at fracture and  $a$  is the crack length. Solving for the fracture stress yields

$$\sigma_{\infty} = \frac{K_{Ic}}{Q\sqrt{\pi a}} = \frac{27MPa\sqrt{m}}{1.12\sqrt{3.14 \times 0.005m}} = 192MPa \quad (2)$$

Thus, fast fracture occurs at  $192MPa$ . Note that this is the far field stress. In order to determine when large-scale yielding occurs, we need to calculate the net stress at the uncracked ligament on the crack plane,  $\sigma_{nom}$ , which is higher than the far field stress,  $\sigma_{\infty}$ , since the net section area on the crack plane,  $A_{net}$  is smaller than the nominal area away from the crack plane,  $A_{\infty}$ . However, given that the plate is large, it can be assumed that the nominal and net section areas are roughly the same. Therefore, when the far field stress,  $\sigma_{\infty}$  is  $192MPa$ , the nominal stress  $\sigma_{nom}$  is roughly equal to that, which is much lower than the yield strength of the material. Thus, the plate will fail by brittle fracture before large scale yielding occurs.

### Problem 3

From Figure 8.36 (p326 Dowling), the fracture toughness for the lowest operating temperature for this problem is  $\sim 35MPa\sqrt{m}$ . In general

$$\sigma_b = \frac{K_I}{Q\sqrt{\pi a}} \leq \frac{K_{Ic}}{Q\sqrt{\pi a}} = \sigma_b^{(frac)} \quad (3)$$

where  $\sigma_b$  is the applied stress in the flange. Thus, failure will occur at

$$\sigma_b = \sigma_b^{(frac)} = \frac{K_{Ic}}{Q\sqrt{\pi a}} = \frac{35MPa\sqrt{m}}{1.12\sqrt{3.14 \times 0.015m}} = 144MPa \quad (4)$$

$Q$  can be safely assumed to be  $Q = 1.12$  since the cracked flange can be treated as an edge-cracked tension member and  $a/w \leq 0.3$ . To account for the safety factor, we simply need to divide the stress result by the factor. Thus,  $\sigma_b^* = \frac{\sigma_b}{1.5} = 96MPa$ . This is the maximum stress in the cracked flange that can be permitted while retaining a safety factor of at least 1.5 with respect to fracture. The area moment of inertia,  $I_x$  for this shape is given as  $I_x = 1.29 \times 10^{-4}m^4$  or it can be manually calculated from page 782 (Dowling). Thus, the maximum permissible bending moment is

$$M^* = \frac{\sigma_b^* I_x}{y} = \frac{96MPa \cdot 1.2910^{-4}m^4}{.152m} = 81.5kNm \quad (5)$$

The maximum moment permitted by the standard design code against yielding is significantly higher than the one calculated against brittle fracture which suggests that the structural member will fail by brittle fracture. This is due to the relatively large crack which would normally be detected by standard inspection techniques.

#### Problem 4

The problem is about an edge crack in a long strip with given width and thickness, subjected to a cyclic bending moment. The bar will fail at the peak tensile stress, which is related to the maximum bending moment by

$$\sigma_b = \frac{M_{(max)}y}{I} = \frac{6M_{(max)}}{tw^2} = 555MPa \quad (6)$$

The critical fracture toughness,  $K_c$  can now be calculated from

$$c = Q\sigma_b\sqrt{\pi a_f} = 1.12 \times 555MPa\sqrt{\pi \times .014m} = 130MPa\sqrt{m} \quad (7)$$

In order for small-scale yielding conditions to apply, the following quantity needs to be evaluated

$$\frac{1}{2\pi} \left( \frac{K_c}{\sigma_y} \right)^2 = 1.7mm \quad (8)$$

and the following to hold

$$a, (w - a), h \geq \frac{15}{2\pi} \left( \frac{K_{Ic}}{\sigma_y} \right)^2 = 26mm \quad (9)$$

$h$  and  $(w - a)$  are adequately large to satisfy the above condition. However, the ratio  $a/\frac{1}{2\pi} \left( \frac{K_c}{\sigma_y} \right)^2 = 8$  and thus the plastic zone at fracture is only  $\sim 1/8$  of the crack size and LEFM and SSY are close.

$a$  is less than  $26mm$  and therefore final failure doesn't take place under small-scale yielding conditions.

Similarly, in order for plane strain conditions to apply, the requirement is that

$$t \geq \frac{15}{2\pi} \left( \frac{K_{Ic}}{\sigma_y} \right)^2 = 26mm \quad (10)$$

The ratio  $t/\frac{1}{2\pi} \left( \frac{K_c}{\sigma_y} \right)^2 = 7$ . Although it doesn't satisfy the plain strain condition, the ratio is large enough to suggest that the results should be close to LEFM.