Proof Let $\bar{b} = \{b_i : i<\lambda\}$ be an enumeration of B.

Let $r(\bar{x}, y) = tp(\bar{b}, a)$. Let $E(y, y') := [\exists \bar{x}\, r(\bar{x}, y) \wedge r(\bar{x}, y')] \vee y = y'$

Then E is a type-definable equivalence relation.

Also: $a E a' \iff B =$

Enumerate all formulas $\varphi(x, x') \vdash x \neq x'$ (ie $T \vdash \forall x \neg \varphi(x, x)$). Enumerate them as $\{\varphi_i(x, x') : i < \lambda\}$.

For every $i < \lambda$ $\exists n_i < \omega$ st:

1. $\exists x_j$ for $j < n_i$ st. $\bigwedge_{j<n_i} p(x_j, a) \wedge \bigwedge_{j<k<n_i} \varphi_i(x_j, x_k)$.
2. $\neg$ " n_i+1 st. " n_i+1 " n_i+1 "

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(Since with ω this is inconsistent, so let n_i be maximal such that it is.)

$$E(y, y') = (y = y') \vee \Big(\bigwedge_{i<\lambda} \exists x_0 \cdots x_{n_i} \bigwedge_{j<n_i} p(x_j, y) \wedge p(x_j, y') \wedge \bigwedge_{j<k<n_i} \varphi_i(x_j, x_k) \Big\} \wedge y, y' \models tp(a) \Big)$$

Clearly: if $a' \models tp(a)$ and $B = \{b : p(b, a')\}$ then $a E a'$.

Now prove converse.

Conversely, assume $a E a'$. So $a' \models tp(a)$.

* IOU rest of proof later...