18.950 Differential Geometry
 Fall 2008

18.950 Homework 9

1. (4 points) Prove that a torus can't have Gauss curvature which is everywhere ≥ 0 (you may use the answers to the second midterm).

2. (6 points) Take $M = S^2$ to be the standard sphere. Find explicitly a moving frame with singularities on M, which (i) has exactly two singularities of multiplicity 1, and (ii) has exactly one singularity of multiplicity 2.

3. (4 points) Let $M = S^2$ again, and consider the function $\phi(y_1, y_2, y_3) = y_1^2$. Determine
$$\int_{S^2} \phi(y) \, dvol_y.$$

4. (6 points) Prove Lemma 28.3 from the lecture notes.