18.336 Numerical Methods of Applied Mathematics -- II
Spring 2009

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Problem Set 4

Out Thu 04/02/09 Due Thu 04/16/09

Problem 8
Consider the linear advection equation

$$\phi_t + u\phi_x + v\phi_y = 0 \ , \quad (x,y) \in]0,1[^2, t \in]0,T[$$

with the velocity field $(u,v) = (-\psi_y, \psi_x)$ where $\psi(x) = \cos(\pi t/T)\sin^2(\pi x)\sin^2(\pi y)$, with $T = 2$. Consider initial conditions $\phi(x,y,0) = (x-0.25)^2 + (y-0.3)^2 - 0.1^2$.

1. Implement an upwind scheme, Lax-Friedrichs, and Lax-Wendroff, and perform a numerical error analysis as with $\Delta t \propto \Delta x$.
2. Design a finite difference approximation that is third order accurate in space. Use (A) a forward Euler step in time, and (B) a third order accurate time step.
3. Investigate stability of your numerical two schemes (A) and (B) using von Neumann stability analysis. If either turns out to be unstable, go back to 2.
4. Perform a numerical error analysis for the schemes (A) and (B), for (a) $\Delta t \propto \Delta x$, and (b) $\Delta t \propto \Delta x^3$. How does your scheme (B) compare with the classical schemes from 1.?
5. Implement a WENO scheme (see lecture) for the problem, and add it to the numerical analysis. How does it perform in comparison to your scheme (B)?
6. Design and program an approach that solves the advection problem with spectral accuracy in the spatial variable. Use a high order time stepping scheme and test the numerical accuracy of your method.