

Chapter 1

1.1 Inner Product of Vectors

Definition: For vectors $x, y \in \mathbb{C}^n$

$$\langle x, y \rangle = x^* y = \sum_{i=1}^n \bar{x}_i y_i \quad (1.1)$$

The inner product is bilinear:

$$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle \quad (1.2)$$

$$\langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle \quad (1.3)$$

$$\langle \alpha x, \beta y \rangle = \bar{\alpha} \beta \langle x, y \rangle \quad (1.4)$$

1.2 Orthogonal vectors

$$\langle x, y \rangle = 0 \quad (1.5)$$

Orthonormal if $\|x\| = \|y\| = 1$.

1.3 Length of a Vector

$$\|x\|_2 = \sqrt{\langle x, x \rangle} = \sqrt{\sum_{i=1}^n |x_i|^2} \quad (1.6)$$