# 18.100B : Fall 2010 : Section R2
# Homework 9

Due Tuesday, November 9, 1pm

**Reading:** Tue Nov. 2 : differentiability, mean value theorem, Rudin 5.1-11
Thu Nov.4 : l'Hospital's rule, Taylor's theorem, Rudin 5.12-19

**1 .** Assume that $f : \mathbb{R} \longrightarrow \mathbb{R}$ and that for some $C > 0$ and $\alpha > 0$ we have for any $x, y \in \mathbb{R}$

$$|f(x) - f(y)| \leq C|x - y|^{\alpha}.$$

**(a)** Prove that if $\alpha > 1$ then $f$ is constant.
[*Hint:* What is the derivative of a constant function?]

**(b)** If $\alpha \leq 1,$ is $f$ necessarily differentiable?

**2 .** Problem # 2 page 114 in *Rudin*.

**3 . (a)** Problem # 7 page 114 in *Rudin*.

**(b)** Show that for any polynomial $P(x)$

$$\lim_{x\to\infty} \frac{P(x)}{e^x} = 0 \qquad \text{and} \qquad \lim_{x\to\infty} \frac{\ln(x)}{P(x)} = 0.$$

For the second limit (of course) assume that $P(x)$ is not constant. You may also use your calculus knowledge of derivatives of polynomials, $e^x$, and $\ln(x)$.

**4 . (a)** Show that $\sin(x) \simeq x$ is a good approximation for small $x$ by using Taylor's theorem to obtain

$$|\sin(x) - x| \leq \tfrac{1}{6}|x|^3 \qquad \forall x \in \mathbb{R}.$$

**(b)** Use (a) to calculate the limit for different values of $a \in \mathbb{R}$ and $c > 0$ of the function $x^a \sin(|x|^{-c})$ (from Rudin pg.115 #13) as $x \to \infty$.

**5 . (a)** Assume $f : (0,1] \longrightarrow \mathbb{R}$ is differentiable and $|f'(x)| \leq M$ for all $x \in (0,1]$. Define the sequence $a_n = f(1/n)$ and prove that $a_n$ converges.

**(b)** Problem # 26 page 119 in *Rudin*.