Preparing Lectures

- Explore, don't merely comprehend An important step in understanding mathematics is to find your own way of thinking about it. Reading and comprehending the text is a necessary first step but you will truly understand the material when your approach to it does not slavishly follow someone else's. Try putting the book away, and asking yourself some basic question about what you've just read - it is by working with the mathematics that you come to own it. Check out the exercises; they often have good ideas to explore.
- Work out examples 'Playing' with examples is the most effective way of making abstract notions concrete. Some textbooks don't emphasise examples in their text, or mention them but don't analyse them fully. Don't be fooled by this; examples are crucially important. The reason they may not be fully explained is that the best approach is to work them out for yourself: you will learn much more that way than by reading someone else's explanation.
- Structure your presentation So now you've read the text, and maybe a few other sources as well, and you've worked out as many examples as you can think of. It's time to take stock of the big picture. What is the main point of what you've learnt? Try to state the big picture in one or two sentences. Then break your new knowledge down into several main points, put in an order so that at each stage you have already developed whatever tools are needed. Then break each of these points down further, to about the level of detail where you are listing the statements of your lemmata. Decide what examples you will use, and what role they will play: will they illustrate theory you've just explained, or motivate a definition or theorem? Don't simply follow the book - after coming to understand the material yourself you may conclude that your way of presenting it is exactly what Osserman has chosen, and that is fine (he has very good taste, after all!) but that is a conclusion you should reach on your own, after understanding things in your own way.
- **Choose your medium** You can use the blackboard, slides, a powerpoint-type presentation, or some combination (you might want, for instance, to give a blackboard talk but incorporate pictures you've downloaded from a website). Blackboard presentations are more common in mathematics than in many other disciplines, and are by far the most common choice for undergraduate and graduate math lectures, although seminar talks increasingly use other media. In a mathematics lecture it is very important that the lecturer and the audience are both thinking through the material as it is presented; the lecturer needs to not be simply reading from lecture notes/slides, but recreating them, at the right pace for the audience. The blackboard forces this upon you more than the other media do, because you are writing the material rather than reading it. I feel that it

is easier to give a 'bad' slide/powerpoint lecture than a 'bad' blackboard one as one can succumb to the temptation of trying to present more than the audience can absorb, or of simply reading one's slides. However, I feel it's about as difficult to give a great talk in any of these media, and in a seminar situation where you probably want to present results without giving all the details about them, the slide/powerpoint presentations come in to their own. Also, in either a lecture or a seminar talk, doing a powerpoint presentation allows you to seamlessly incorporate things like videos or interactive java applets that can be wonderful additions to your presentation.

- Blackboard talk: Make detailed lecture notes Now it's time to expand your outline out into a lecture. What it's appropriate to write on the board is different from what you would write if you were writing a textbook. Be succinct. Avoid long sentences. You should write out lecture notes that consist of what you intend to write on the board. Some people say that you should write out notes *exactly* as you plan to present the material; the same number of lines as you'll put on each board etc (and work out how many lines should go on each board before the class). You may feel that this is taking things too far, but it is important to at least produce notes that accurately represent what you will do in the lecture. It's also helpful to make a note of any additional comments that you plan to make, but anything important (for instance anything that made it on to your outline) should be written down on the board. You may want to expand on some side issue without writing down the details, or mention that something you've just introduced will be used again later, but in general at least a summary of what you're saying should make it on to the board.
- Slides/Powerpoint Presentation Make sure that your font sizes are large enough that the audience will easily be able to read your text (this is especially important if you're in a large lecture hall), and don't crowd each page - use double spacing, and an attractive, uncramped layout. Colours are great, too.
- **Do a Timed Practice** Do a 'dry run' on the medium you plan to use, and time it. If there's someone else who's interested in listening, that's great but it's not necessary. Time yourself, and take note not only of the total time but of the time it takes you to get to a few different points in your lecture, so that on the day you'll know whether you are running a bit fast or a bit slow. Plan how you can adjust in each case; something extra you can say or something you can say more succinctly or cut out altogether.

Giving Lectures

0.1 Blackboard Talk

- **Summarise** It's often a good idea to give your audience a quick summary of what you are going to say, so that they know what to expect and they recognise the main points immediately.
- **Give it live** You don't want your lecture to consist of you copying your notes on to the board, and faithfully reading them to us. You should be able to remember large parts of your lecture; you will want your notes close by and you'll probably glance at them often to make sure you're giving the lecture as planned, but make sure that you are actively understanding and explaining the material as you present it. When you come to a proof or example this is particularly important or you'll get lost.
- Use the boards in order Start by erasing anything that the previous class left on the board. Begin writing on the left with one of the movable boards, then push it up and write on the bottom left board, and work your way rightwards, always making sure that you first write on a board that can be pushed up. For wide boards you may want to draw a line down the middle to divide it into smaller pieces.
- Make your writing legible Neat writing, at a size that is easy to read (but not so big you can't fit much on the board) is one of those things one comes to appreciate when it's missing. Don't try to cram something in to a tiny piece of the board: begin a new one, if you need to. These last two points are easy to do, and they make a big difference.
- Repeat, repeat, repeat You know what is is like to be in a lecture no matter how interesting the lecture is, you'll space out for a few seconds occasionally. As a lecturer, you need to make sure that a few moments of inattention don't make it impossible for someone to follow what you say next. This is one of the reasons for writing down some account of pretty much everything you say. You should also talk about what you're writing, and since it's quicker to speak than to write you have time to emphasise the main point of each paragraph by repeating it. This tells people that it is important and gives them another chance to understand your point, especially if you can say it in a slightly different way.
- Look at the audience Try not to talk to the blackboard; you'll need to face it whilst writing but turn around as much as possible, and spend some time not writing anything so that you can engage with the audience. Try to make eye contact with each person in the room at some point during your lecture; don't just direct your remarks to the person who asked an interesting question at the beginning of class.
- **Encourage class participation** A lively class in which people are asking questions, and actively contributing, is a lot more fun and a better place to

learn. When it's not your turn to lecture, you can help create such a classroom by asking and responding to questions. When you are lecturing, you can try to draw people in to a discussion by asking them questions, and by setting up some short, easy task that everyone can work through, computing out an example for instance. This means that people are immediately applying what you are teaching them and hence they understand it and are really ready for the next thing that you introduce. It also keeps everyone interested; it's hard to sit in a long lecture; taking a break to *do* some mathematics makes the class more enjoyable.