Review for Exam 1
18.05 Spring 2014

Normal Table

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970

Topics

1. Sets.
2. Counting.
3. Sample space, outcome, event, probability function.
4. Probability: conditional probability, independence, Bayes' theorem.
5. Discrete random variables: events, pmf, cdf.
6. Bernoulli(p), binomial(n, p), geometric(p), uniform(n)
7. $E(X)$, Var(X), σ
8. Continuous random variables: pdf, cdf.
9. uniform(a,b), exponential(λ), normal(μ,σ^2)
10. Transforming random variables.
11. Quantiles.
12. Central limit theorem, law of large numbers, histograms.
13. Joint distributions: pmf, pdf, cdf, covariance and correlation.

Sets and counting

- Sets:
  $\emptyset$, union, intersection, complement Venn diagrams, products
- Counting:
  inclusion-exclusion, rule of product, permutations ${}_nP_k$, combinations ${}_nC_k = \binom{n}{k}$

Probability

- Sample space, outcome, event, probability function.
  Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
  Special case: $P(A^c) = 1 - P(A)$
  (A and B disjoint $\Rightarrow P(A \cup B) = P(A) + P(B)$.)
- Conditional probability, multiplication rule, trees, law of total probability, independence
- Bayes' theorem, base rate fallacy

Random variables, expectation and variance

- Discrete random variables: events, pmf, cdf
- Bernoulli(p), binomial(n, p), geometric(p), uniform(n)
- $E(X)$, meaning, algebraic properties, $E(h(X))$
- Var(X), meaning, algebraic properties
- Continuous random variables: pdf, cdf
- uniform(a,b), exponential(λ), normal(μ,σ)
- Transforming random variables
- Quantiles

Central limit theorem

- Law of large numbers averages and histograms
- Central limit theorem

Joint distributions

- Joint pmf, pdf, cdf.
- Marginal pmf, pdf, cdf
- Covariance and correlation.

Hospitals (binomial, CLT, etc)

- A certain town is served by two hospitals.
- Larger hospital: about 45 babies born each day.
- Smaller hospital about 15 babies born each day.
- For a period of 1 year, each hospital recorded the days on which more than 60% of the babies born were boys.

(a) Which hospital do you think recorded more such days?
(i) The larger hospital. (ii) The smaller hospital.
(iii) About the same (that is, within 5% of each other).

(b) Assume exactly 45 and 15 babies are born at the hospitals each day. Let L_i (resp., S_i) be the Bernoulli random variable which takes the value 1 if more than 60% of the babies born in the larger (resp., smaller) hospital on the i^{th} day were boys. Determine the distribution of L_i and of S_i.

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Hospital continued

(c) Let L (resp., S) be the number of days on which more than 60% of the babies born in the larger (resp., smaller) hospital were boys. What type of distribution do L and S have? Compute the expected value and variance in each case.

(d) Via the CLT, approximate the 0.84 quantile of L (resp., S). Would you like to revise your answer to part (a)?

(e) What is the correlation of L and S? What is the joint pmf of L and S? Visualize the region corresponding to the event $L > S$. Express $P(L > S)$ as a double sum.

Problem correlation

1. Flip a coin 3 times. Use a joint pmf table to compute the covariance and correlation between the number of heads on the first 2 and the number of heads on the last 2 flips.

2. Flip a coin 5 times. Use properties of covariance to compute the covariance and correlation between the number of heads on the first 3 and last 3 flips.