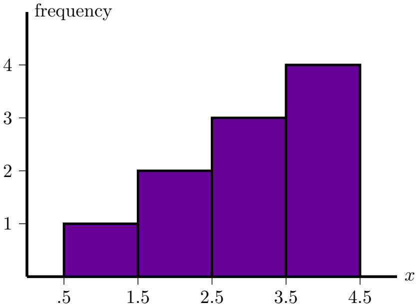
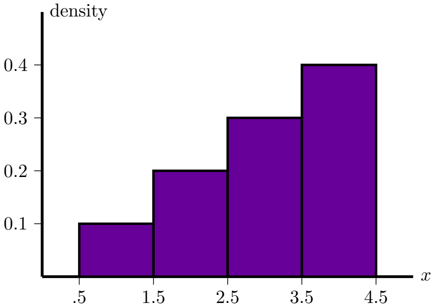
Studio 3

18.05 Spring 2014

Jeremy Orloff and Jonathan Bloom





Concept questions

Suppose X is a continuous random variable.

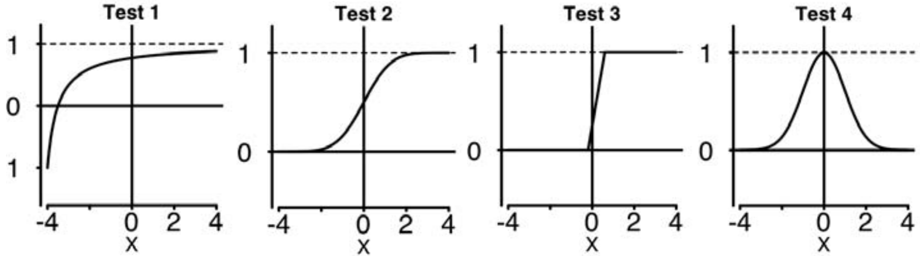
a) What is $P(a \leq X \leq a)$?

b) What is $P(X = 0)$?

c) Does $P(X = 2) = 0$ mean X never equals 2?

Concept question

Which of the following are graphs of valid cumulative distribution functions?



Add the numbers of the valid cdf's and click that number.

Exponential Random Variables

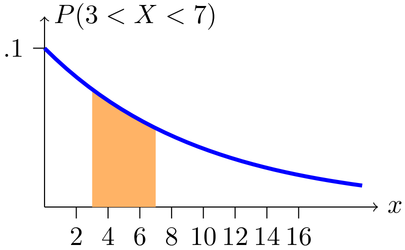
Parameter: λ (called the rate parameter).

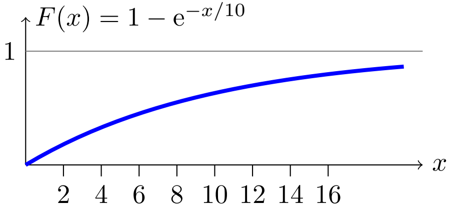
Range: $[0, \infty)$.

Notation: exponential(λ) or exp(λ).

Density: $f(x) = \lambda e^{-\lambda x}$ for $0 \le x$.

Models: Waiting time





Continuous analogue of geometric distribution –memoryless!

Uniform and Normal Random Variables

Uniform: $U(a, b)$ or uniform(a, b)
Range: $[a, b]$
PDF: $f(x) = \dfrac{1}{b - a}$

Normal: $\mathrm{N}(\mu, \sigma^2)$
Range: $(-\infty, \infty]$
PDF: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(x-\mu)^2/2\sigma^2}$

http://mathlets.org/mathlets/probability-distributions/

Table questions

Open the applet

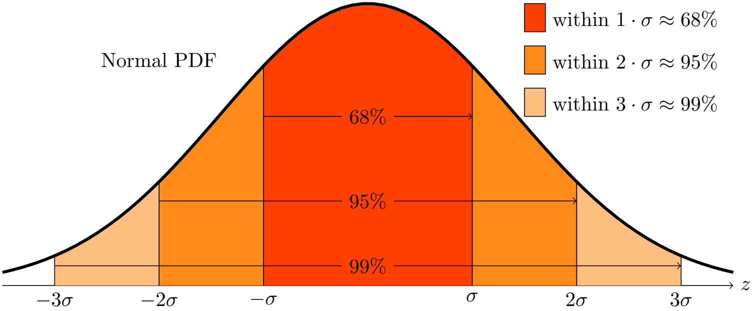
http://mathlets.org/mathlets/probability-distributions/

1. For the **standard normal** distribution $N(0, 1)$ how much probability is within 1 of the mean? Within 2? Within 3?

2. For $N(0, 3^2)$ how much probability is within σ of the mean? Within 2σ? Within 3σ.

3. Does changing μ change your answer to problem 2?

Normal probabilities



Rules of thumb:

$P(-1 \leq Z \leq 1) \approx .68,$
$P(-2 \leq Z \leq 2) \approx .95,$
$P(-3 \leq Z \leq 3) \approx .997$

Download R script

Download studio3.zip and unzip it into your 18.05 working directory.

Open studio3.r in RStudio.

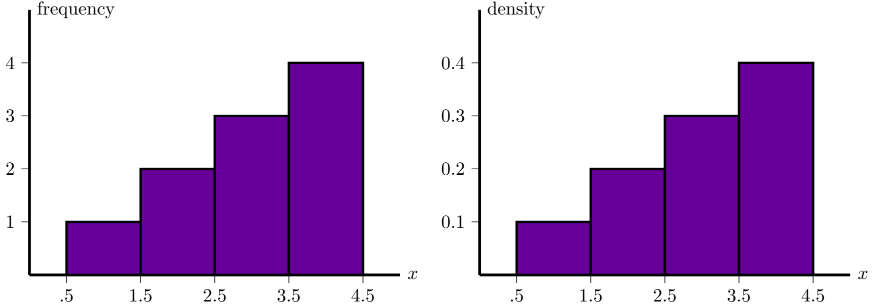
Histograms

Will discuss in more detail in class 6.

Made by 'binning' data.

Frequency: height of bar over bin = # of data points in bin.

Density: area of bar over bin is proportional to # of data points in bin. Total area of a density histogram is 1.



Histograms of averages of exp(1)

1. Generate a frequency histogram of 1000 samples from an exp(1) random variable.

2. Generate a density histogram for the average of 2 independent exp(1) random variable.

3. Using `rexp()`, `matrix()` `and` `colMeans()` generate a density histogram for the average of 50 independent exp(1) random variables. Make 10000 sample averages and use a binwidth of .1 for this. Look at the spread of the histogram.

4. Superimpose a graph of the pdf of $N(1, 1/50)$ on your plot in problem 3. (Remember the second parameter in N is σ^2.)

18.05 Introduction to Probability and Statistics
Spring 2014