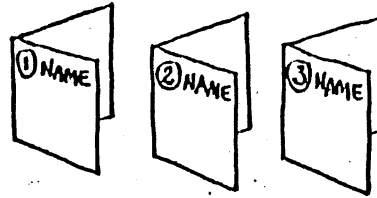


CLOSED BOOK ... and NO calculators

As before, please struggle with these problems on separate sheets of paper ...



- 1) Not necessarily via residue calculus, please evaluate

$$\int_0^{\infty} \frac{\ln x}{1+x^2} dx$$

HINTS: Perhaps try  ... or just some substitution?

- 2) a) Which region of the complex z -plane gets mapped by

$$w = \frac{z-1}{z+1}$$

into the interior of the circle $|w| = 1$, and why?

- b) Use the above answer as a clue to find a related bilinear transformation $W(z)$ that carries the top half of the z -plane into a unit circle centered instead at $W = 1 + i$.

- 3) For that "steady-state" solution of $\frac{dx}{dt} + x = \cos \omega t$ which can be written as

$$x(t) = \text{Re} \{ A(\omega) e^{i\omega t} \}$$

show that the amplitude A , regarded as a function of the real "forcing frequency" ω , traces out a circle in the complex A -plane.