18.03 Problem Set 5, First Half

The second half will be available by Monday, March 29.

I encourage collaboration on homework in this course. However, if you do your homework in a group, be sure it works to your advantage rather than against you. Good grades for homework you have not thought through will translate to poor grades on exams. **You must turn in your own writeups of all problems, and, if you do collaborate, you must write on the front of your solution sheet the names of the students you worked with.**

Because the solutions will be available immediately after the problem sets are due, **no extensions will be possible**.

II. Second-order linear equations

L17	F 12 Mar	LTI systems, superposition, review.
L18	M 15 Mar	Engineering examples: an interview with Prof. Kim Vandiver: Damping ratio, review.
R12	T 16 Mar	Exam review.
L19	W 17 Mar	**Hour Exam II.**
R13	Th 18 Mar	Getting ready for Fourier series
L20	F 19 Mar	Fourier Series: EP 8.1
L21	M 29 Mar	Operations on Fourier Series: EP 8.2, 8.3.
R14	T 30 Mar	Ditto.
L22	W 31 Mar	Periodic solutions, resonance: EP 8.3, 8.4.
R15	Th 1 Apr	Ditto.
L23	F 2 Apr	Step function and delta function: SN 17.

Part I.

17. (F 12 Mar) EP 2.5: 8. Also: Find a solution to $\ddot{x} + x = t^2 + \cos(2t - 1)$.

18. (M 15 Mar) Nothing new.

19. (W 17 Mar) Hour exam.

Part II.

17. (F 12 Mar) [Series RLC circuits; amplitude and phase] Open the Mathlet `Series RLC Circuit`. This applet is used also in 8.02, where the voltage drops are important. We will focus entirely on the current response, so it will be clearer if the check boxes labelled V_R, V_L, V_C, are left unchecked. But click twice on the I box, to make a green curve appear in the graphing window, representing the current through any point in the circuit as a function of time.

The Mathlet uses the International System of Units, SI, formerly known as the mks (meter-kilogram-second) system. The equation

$$L\ddot{I} + R\dot{I} + (1/C)I = \dot{V}$$

is correct when:
the resistance R is measured in ohms, Ω,
the inductance L is measured in H, henries,
the capacitance C is measured in farads, F,
the voltgage V is measured in volts, also denoted V,
the current I is measured in amperes, A.

The slider displays millihenries, mH (1 mH= 10^{-3} H) and microfarads, μF (1μF= 10^{-6} F), and milliseconds, ms (1 ms = 10^{-3} sec).

The Mathlet studies a sinusoidal input signal $V(t) = V_0 \sin(\omega t)$. Play around with the various sliders and watch the effect on the (blue) voltage curve and the (green) current curve.

(a) By experimenting, identify a few values of the system parameters R, L, C, V_0, ω, for which the current and the voltage are perfectly *in phase*. For example, if $L = 500$ mH and $\omega = 200$ radians/second, what values of R, C, and V_0 put I in phase with V?

(b) Now calculate the relationship between the system parameters which leads to I and V being in phase. Do your experiments align with your calculations?

(c) Set $R = 100\,\Omega$, $L = 1000$ mH, $C = 100\,\mu$F, $V_0 = 500$ V. Vary ω and watch the action. For what value of ω is the amplitude of $I(t)$ maximal? What is that amplitude (in amps)? What is the phase lag between the input signal, $V_0 \sin(\omega t)$, and the system response, $I(t)$, for that value of ω?

(d) Verify the three observations made in **(c)** computationally. You should be able to do this for general values of R, L, C, V_0.

18. (M 15 Mar) Short problem on damping ratio: We called $\omega_n = \sqrt{k/m}$ the *natural frequency* of the system modeled by the homogeneous equation $m\ddot{x}+b\dot{x}+kx = 0$. Critical damping occurs at $b/m = 2\omega_n$. Write b/m as a multiple of critical damping: $b/m = 2\omega_n\zeta$. The system is overdamped if the "damping ratio" $\zeta > 1$, underdamped if $\zeta < 1$.

Suppose the system is underdamped, and that a solution is observed to decrease in amplitude by a factor of two (that is, the exponential factor in the solution decreases to half its value at $t = 0$) after n cycles. Calculate ζ in terms of n and the constant $\alpha = \ln 2/2\pi \simeq 0.1103178$. If $n = 10$, what is ζ? Compare this exact value to the engineer's approximation α/n.

19. (W 10 Mar) Hour exam.

Part I answers. 17. $x = (t^2 - \frac{1}{2}) - \frac{1}{3}\cos(2t - 1)$.

18.03 Differential Equations
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