## Problems: Normal Form of Green's Theorem

Use geometric methods to compute the flux of $\mathbf{F}$ across the curves $C$ indicated below, where the function $g(r)$ is a function of the radial distance $r$.

1. $\mathbf{F}=g(r)\langle x, y\rangle$ and $C$ is the circle of radius $a$ centered at the origin and traversed in a clockwise direction.
Answer: (Radial field) $\mathbf{F}$ is parallel to $\mathbf{n}$ with $\langle x, y\rangle=a \mathbf{n}$ on $C$, so we have $\mathbf{F} \cdot \mathbf{n}=g(a) \cdot a$ $\Rightarrow$ Flux $=g(a) 2 \pi a^{2}$.
2. $\mathbf{F}=g(r)\langle-y, x\rangle ; C$ as above.

Answer: (Tangential field) Since $\mathbf{F}$ is orthogonal to $\mathbf{n}$ the flux is 0 .
3. $\mathbf{F}=3\langle 1,1\rangle ; C$ is the line segment from $(0,0)$ to $(1,1)$.

Answer: Since $\mathbf{F}$ is parallel to the line segment $C$ we have $\mathbf{F} \cdot \mathbf{n}=0 . \Rightarrow$ flux $=0$.
4. $\mathbf{F}=3\langle-1,1\rangle ; C$ is the line segment from $(0,0)$ to $(1,1)$.

Answer: $\mathbf{F}$ is orthogonal to $C$. $\mathbf{F}$ points in the opposite direction from $\mathbf{n}$ because $\mathbf{n}$ is clockwise from the direction vector for $C$.
$\Rightarrow$ flux $=\int \mathbf{F} \cdot \mathbf{n} d S=\int 3 \sqrt{2} d s=6$.

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