Problems: Normal Form of Green's Theorem

Use geometric methods to compute the flux of \mathbf{F} across the curves C indicated below, where the function g(r) is a function of the radial distance r.

1. $\mathbf{F} = g(r)\langle x, y \rangle$ and *C* is the circle of radius *a* centered at the origin and traversed in a clockwise direction.

<u>Answer</u>: (Radial field) **F** is parallel to **n** with $\langle x, y \rangle = a\mathbf{n}$ on *C*, so we have $\mathbf{F} \cdot \mathbf{n} = g(a) \cdot a$ \Rightarrow Flux = $g(a)2\pi a^2$.

2. $\mathbf{F} = g(r)\langle -y, x \rangle$; *C* as above.

<u>Answer:</u> (Tangential field) Since \mathbf{F} is orthogonal to \mathbf{n} the flux is 0.

3. $\mathbf{F} = 3\langle 1, 1 \rangle$; C is the line segment from (0, 0) to (1, 1).

<u>Answer</u>: Since **F** is parallel to the line segment C we have $\mathbf{F} \cdot \mathbf{n} = 0$. \Rightarrow flux = 0.

4. **F** = $3\langle -1, 1 \rangle$; *C* is the line segment from (0,0) to (1,1).

<u>Answer:</u> **F** is orthogonal to C. **F** points in the opposite direction from **n** because **n** is clockwise from the direction vector for C.

 \Rightarrow flux = $\int \mathbf{F} \cdot \mathbf{n} \, dS = \int 3\sqrt{2} \, ds = 6.$

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