Functions of two variables

Examples: Functions of several variables
$f(x,y) = x^2 + y^2 \;\Rightarrow\; f(1,2) = 5$ etc.
$f(x,y) = xy^2\mathrm{e}^{x+y}$
$f(x,y,z) = xy\log z$
Ideal gas law: $P = kT/V$.

Dependent and independent variables
In $z = f(x,y)$ we say x, y are independent variables and z is a dependent variable. This indicates that x and y are free to take any values and then z depends on these values. For now it will be clear which are which, later we'll have to take more care.

Graphs
For the function $y = f(x)$: there is one independent variable and one dependent variable, which means we need 2 dimensions for its graph.

Graphing technique:

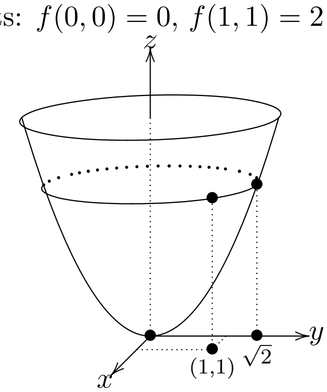
go to x then compute $y = f(x)$ then go up to height y.

For $z = f(x,y)$ we have two independent and one dependent variable, so we need 3 dimensions to graph the function. The technique is the same as before.

Example: Consider $z = f(x,y) = x^2 + y^2$.
To make the graph:

go to (x,y) then compute $z = f(x,y)$ then go up to height z.

We show the plot of three points: $f(0,0) = 0$, $f(1,1) = 2$ and $f(0,\sqrt{2}) = 2$.



The figure above shows more than just the graph of three points. Here are the steps we used to draw the graph. Remember, this is just a sketch, it should suggest the shape of the graph and some of its features.

1. First we draw the axes. The z-axis points up, the y-axis is to the right and the x-axis comes out of the page, so it is drawn at the angle shown. This gives a perspective with the eye somewhere in the first octant.

2. The yz-traces are those curves found by setting $x =$ a constant. We start with the trace when $x = 0$. This is an upward pointing parabola in the yz-plane.

3. Next we sketch the trace with $z = 3$. This is a circle of radius $\sqrt{3}$ at height $z = 3$. Note, the traces where $z =$ constant are generally called *level curves*.

This is enough for this graph. Other graphs take other traces. You should expect to do a certain amount of trial and error before your figure looks right.