

SECOND MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name: _____

Signature: _____

Recitation Time: _____

There are 5 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20pts) Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ be the function given by $f(x, y, z) = x^3y + y^3z + z^3x - 2xyz$.

(i) Find the gradient of f at $P = (2, -1, 1)$.

(ii) Find the directional derivative of f at P in the direction of $\hat{u} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$.

(iii) Find the tangent plane at the point P of the level surface $\{Q \in \mathbb{R}^3 \mid f(Q) = -3\}$.

2. (20pts) Suppose that $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is differentiable at $P = (-1, 4)$ with derivative

$$DF(-1, 4) = \begin{pmatrix} -1 & 1 \\ 3 & -2 \\ -2 & -1 \end{pmatrix}.$$

Suppose that $F(-1, 4) = (1, -1, 3)$. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function $f(x, y) = \|F(x, y)\|$.

(i) Show that the function $f(x, y)$ is differentiable at P .

3. (20pts) Let

$$S = \{ (x, y, z) \in \mathbb{R}^3 \mid x^3y + y^2z^3 + zx^2 = 3 \}.$$

(i) Show that S is the graph of a function $z = f(x, y)$ in a neighbourhood of $P = (1, -2, 1)$.

4. (20pts) Let $\vec{r}: I \rightarrow \mathbb{R}^3$ be a regular smooth curve parametrised by arclength. Let $a \in I$ and suppose that

$$\vec{N}(a) = \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}, \quad \vec{B}(a) = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}, \quad \frac{d\vec{N}}{ds}(a) =$$

Find:

(i) $\vec{T}(a)$.

(ii) $\kappa(a)$

(iii) $\tau(a)$

5. (20pts) Let $\vec{F}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the vector field given by $f(x, y) = y\hat{i} - 2\hat{j}$.

(i) Is \vec{F} a gradient field (that is, is \vec{F} conservative)? Why?

(iii) Find a flow line that passes through the point (a, b) .

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18.022 Calculus of Several Variables
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