Surface Area of a Wine Glass

Professor Jerison found the volume of a "wine glass" shape formed by revolving the graph of $y = e^x$ $(0 \le x \le 1)$ about the y-axis. Set up but do not evaluate an integral to compute the surface area of that shape.

Solution

The main difference between this problem and the example presented in lecture is that we are revolving about the y-axis, not the x-axis. Hence, we are adding up areas of circular "ribbons" whose radius is x (not y) and whose width is ds.

$$\text{Area} = \int_{x_1}^{x_2} 2\pi x \, ds$$

Recalling that $\dfrac{ds}{dx} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$, we get:

$$\begin{aligned} \text{Area} &= \int_0^1 2\pi x \sqrt{1 + (e^x)^2} \, dx \\ &= \int_0^1 2\pi x \sqrt{1 + e^{2x}} \, dx. \end{aligned}$$

Numerical methods predict that the value of this integral is close to 7. The surface area of a cone whose base is at the lip of the glass and whose tip is at the center of the bottom of the glass is around 6, so our answer appears to be correct.

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