Integral of $\dfrac{x^3}{x^2-1}$

Express the integrand as a sum of a polynomial and a proper rational function, then integrate:

$$\int \frac{x^3}{x^2-1}\,dx.$$

Solution

The numerator of the integrand has a higher degree than the denominator, so we must use long division to convert the integrand from an "improper fraction" to a "mixed fraction".

$$\begin{array}{r} x \\ x^2-1\,\overline{)\;x^3} \\ \underline{-\,x^3+x} \\ x \end{array}$$

We can now write $\dfrac{x^3}{x^2-1} = x + \dfrac{x}{x^2-1}$. (Substituting 2 for x gives us $\frac{8}{3} = 2+\frac{2}{3}$, so this is probably correct.)

The remainder of the calculation is fairly simple, involving one substitution of $u = x^2-1$, $du = 2x\,dx$.

$$\begin{aligned} \int \frac{x^3}{x^2-1}\,dx &= \int x + \frac{x}{x^2-1}\,dx \\ &= \frac{1}{2}x^2 + \frac{1}{2}\ln|u| + c \\ &= \frac{1}{2}x^2 + \frac{1}{2}\ln|x^2-1| + c \end{aligned}$$

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