Long Division

When you're integrating a rational expression $\dfrac{P(x)}{Q(x}$, what happens if the degree of P is greater than or equal to the degree of Q? We can think of this as an improper fraction, similar to fractions like $\frac{5}{4}$ and $\frac{8}{3}$.

Example:

$$\frac{x^3}{(x-1)(x+2)}$$

The numerator has degree 3 and the denominator has degree 2; our usual method is not going to work here.

The first step to simplifying this turns out to be to reverse our usual step 1; we don't want the denominator factored, we want it expanded, or multiplied out:

$$\frac{x^3}{(x-1)(x+2)} = \frac{x^3}{x^2+x-2}.$$

Our next step is to use long division to convert an improper fraction into a proper fraction:

$$\begin{array}{r l}
 & \phantom{-x^3 - {}} x-1 \\ \cline{2-2}
x^2+x-2 \big) & x^3 \\
 & -x^3 - x^2 + 2x \\ \cline{2-2}
 & -x^2 + 2x \\
 & x^2 + x - 2 \\ \cline{2-2}
 & 3x - 2
\end{array}$$

You may recall from grade school that $x-1$ is called the *quotient* and $3x-2$ is the *remainder*. We use this result to rewrite our rational expression as follows:

$$\frac{x^3}{(x-1)(x+2)} = \underbrace{x-1}_{\text{easy}} + \underbrace{\frac{3x-2}{x^2+x-2}}_{\text{use cover-up}}$$

We could now find the integral of this rational expression if we wished to.

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