

Integral of $\tan^3(x)$

Use what you have learned to integrate the function $\tan^3(x)$.

Solution

This is a relatively simple integration; the method described below uses a substitution and the properties $\sec^2 x = 1 + \tan^2 x$ and $\int \tan x dx = -\ln |\cos x| + c$.

$$\begin{aligned}\int \tan^3 x dx &= \int \tan x \tan^2 x dx \quad (\text{use an identity to reduce degree}) \\ &= \int \tan x (\sec^2 x - 1) dx \\ &= \int \underbrace{\tan x}_u \underbrace{\sec^2 x dx}_{du} - \int \tan x dx \\ &= \int u du - (-\ln |\cos x| + c) \\ &= \frac{1}{2}u^2 + \ln |\cos x| + c \\ &= \frac{1}{2} \tan^2 x + \ln |\cos x| + c\end{aligned}$$

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