Example: $\int \cos^2 x\, dx$

What if we have to integrate $\int sin^n x \cos^m x\, dx$ when both exponents are even? This is a harder case; we'll use the half angle formulas to solve it.

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$
$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

These formulas help us by turning even powers of $\sin x$ and $\cos x$ into odd powers of $\cos(2x)$.

If we wanted to integrate:

$$\int \cos^2 x\, dx,$$

we could rewrite it as $\int (1-\sin^2 x)\, dx$, but the new integral is at least as hard as the one we started with. Instead we use a half angle formula:

$$\begin{aligned}\int \cos^2 x\, dx &= \int \frac{1+\cos(2x)}{2}\, dx \\ &= \frac{x}{2} + \frac{\sin 2x}{4} + c\end{aligned}$$

Notice that $\frac{x}{2}$ appears in the solution and is not a trigonometric function!

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