$$\int \cos^3(2x)\,dx$$

Compute $\int \cos^3(2x)\,dx$.

Solution

One could apply double or possibly triple angle formulas to this problem, but it turns out to be fairly simple to compute this integral using the methods Professor Miller outlined in lecture. We start by substituting $u = 2x$ (so $\frac{1}{2}du = dx$). The problem then becomes a straightforward integral of a trigonometric function with an odd exponent, which we solve using the identity $\cos^2(u) = 1 - \sin^2(u)$.

$$\begin{aligned}
\int \cos^3(2x)\,dx &= \frac{1}{2}\int \cos^3(u)\,du \qquad (u = 2x) \\
&= \frac{1}{2}\int \cos^2(u)\cos(u)\,du \\
&= \frac{1}{2}\int (1-\sin^2(u))\cos(u)\,du \\
&= \frac{1}{2}\int (1-v^2)\,dv \qquad (v = \sin u) \\
&= \frac{1}{2}\left(v - \frac{1}{3}v^3 + c\right) \\
&= \frac{1}{2}\left(\sin(2x) - \frac{1}{3}\sin^3(2x) + c\right) \\
\int \cos^3(2x)\,dx &= \frac{1}{2}\sin(2x) - \frac{1}{6}\sin^3(2x) + C.
\end{aligned}$$

We can check this answer by differentiating; be sure to apply the chain rule twice when differentiating $\frac{1}{6}\sin^3(2x)$.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{1}{2}\sin(2x) - \frac{1}{6}\sin^3(2x) + C\right) &= \cos(2x) - \frac{1}{6}\cdot 3\sin^2(2x)\cos(2x)\cdot 2 + 0 \\
&= \cos(2x) - \sin^2(2x)\cos(2x) \\
&= \cos(2x) - (1-\cos^2(2x))\cos(2x) \\
&= \cos(2x) - \cos(2x) + \cos^3(2x) \\
&= \cos^3(2x).
\end{aligned}$$