Numerical Integration Study Tips

How can we remember the formulas for the trapezoidal rule and Simpson's rule to use on the exam? How can we know if we've remembered the formulas correctly?

Instead of memorizing the entire formula, you might memorize a very simple case:

$$f(x) = 1$$

and reconstitute the complete formula from that. The formula for $f(x) = 1$ is easy to check — if the area under the curve doesn't come out to $b - a$, you're doing it wrong.

Consider the formula for the trapezoid rule:

$$\text{Area} \approx \Delta x \left(\frac{1}{2} y_0 + y_1 + \cdots + y_{n-1} + \frac{1}{2} y_n \right).$$

If the function is $f(x) = 1$, then $y_i = 1$ for each value of i:

$$\begin{aligned} \Delta x \left(\frac{1}{2} + \overbrace{1 + 1 + \cdots + 1}^{(n-1) \text{ of these}} + \frac{1}{2} \right) &= \Delta x \left(\frac{1}{2} + (n-1) + \frac{1}{2} \right) \\ &= \Delta x \cdot n \\ &= \frac{b-a}{n} \cdot n \\ &= b - a \\ &= \int_a^b 1 \, dx. \end{aligned}$$

You can do the same thing with Simpson's rule.

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