

PROFESSOR: Hi. Welcome back to recitation. In lecture you've been learning about applications of integration. And one of them is how to compute average values of a function. So I have a nice average value problem for you here.

So Christine was out jogging one day and she saw a bear, and so she broke into a sprint. She starts speeding up, and her velocity t seconds after she started sprinting is given by v of t equals 1500 divided by the quantity 100 plus t minus 5 squared, the whole thing minus 7 . And it's in meters per second.

So OK, so she starts at t equals 0 , she's going 1500 divided by 125 , so that's 12 minus 7 , so 5 meters per second. And she starts to speed up and after 5 seconds she's going 1500 divided-- well this is 0 -- so she's going 8 meters per second at 5 seconds. And then she looks over her shoulder and she sees, oh no, it wasn't up a bear, it was just an unkempt math graduate student, so then she starts to slow down again. She slows back down and 10 seconds later she's back to her original velocity.

So the question is, over the 10 seconds of this sprint, she sped up, she slowed down, from 5 meters per second to 8 meters per second and then back to 5 . What was her average velocity over this time? So why don't you pause the video, take some time to work that out, come back and we can work on it together.

So hopefully you had some luck working on this problem. I have over on my left here a little picture of the graph of this function v of t . I described it to you earlier, but just you see, OK, so it starts at 5 , after 5 seconds it's at 8 , and then it goes back down to 5 at 10 seconds. So this is roughly the picture, so we have a feel for what we're working with. Now what we want to compute is the average velocity over this time.

So anytime you have to compute an average value of any function what you want to do, you always do the same thing. So you want to compute the total contribution of that function over the interval in question. So you want to add it up, you want to integrate it over the interval in question. But then you want to divide by the length of the interval. So in particular, when you have a velocity function what that means is, you integrate the velocity. So that gives you the total distance traveled, and then you divide by the time interval. So you're just taking total distance divided by total time.

So in this case we can write down this average value. So I'm going to use just a made up notation. I'm going to write avg, for average, of v , so what is it? Well the first thing I have to do is I want to not forget to divide by the length of the interval here. So in this case, the interval is from 0 to 10, so it has length 10 minus 0, which is 10. So I want $1/10$ in front and what I want to multiply this by, is I want to multiply by the integral over this entire interval of my function v .

So I just take the formula that I had over there for v and I plop it down into my integral. So it's the integral from 0 to 10 of 1500 divided by 100 plus t minus 5 quantity squared minus $7t$ dt. So this integral is the average value that we're looking for.

OK. So now we have to go about evaluating it. Now, this looks kind of ugly, but actually it's not that bad. I mean, one thing you can notice, so the minus 7 part, that's going to be easy to take care of. And then in this more complicated part, well there's a kind of obvious substitution here. You can set u equal t minus 5 , that'll simplify it a little bit. and then what you'll see is that you have an integrand of the form something divided by 100 plus u squared. And so that's 10 squared plus u squared. So that should keep-- and the something as a constant. So that should put you in the mind of a tangent substitute, or just even not by substitution, you can just remember that that's an arctangent type of thing that you're going to get out.

so I'm not going to go through all the steps of doing that, but we should get at the end-- well OK, so I've got this $1/10$ out in front, so I have to not forget about that. I get $1/10$ times, I've gone ahead and I've computed the anti-derivative and it's 150 times arctan of the quantity t minus 5 over 10 . So here's that t minus 5 , that's the same t minus 5 over there that's coming out. Minus $7t$. And you want to take that between t equals 0 and t equals 10 .

So this is, I've just, I've gone ahead, I've computed the anti-derivative for us. There's my $1/10$ because I'm doing an average value and then I'm, it's a definite integral and so I'm using fundamental theorem of calculus; there are my bounds. OK, so now we just have to plug in and evaluate. So this is equal to, so let's see. So we've got when we put in t equals 10 -- well the $1/10$ and the 150 gives me a 15 -- arctan of $1/2$ minus-- well it's $1/10$ of 7 times 10 , it's just 7 . That's from the first one.

And from the second one I guess minus-- so it's going to be 15 arctan of minus $1/2$. When I put in t equals 0 , and when I put in t equals 0 , the $7t$ is just 0 . OK. This is a little messy. We can simplify it a little bit, because, remember arctangent is an odd function. So arctan of minus x is minus arctangent of x . So minus 15 arctan minus $1/2$ is the same as just 15 arctan $1/2$.

And so we can combine those.

So we can rewrite that. Let's go all the way over here. We can rewrite this as $30 \arctan \frac{1}{2}$ minus 7. So that's about it as nice a form as you can make this take. If you wanted to plug it into a calculator, I think you'd see that this is approximately equal to 6.9 and I guess the units there are going to be meters per second. So her average velocity over this time is about 6.9 meters per second.

Let's just quickly recap what we did. So we started all the way back here with this velocity function v , and we wanted to compute it's average value over the interval $0 \leq t \leq 10$. So we did what we always do in problems of that sort. So we came over here, and whenever you want to compute an average value, you take 1 divided by the length of the interval in question, times-- then just you integrate the function who's average value you want over the interval. OK?

And so then we computed, well we didn't show any of the steps, but we computed the definite integral by taking the anti-derivative and plugging in and it happened that the answer worked out to be about 6.9. So there you go. I'll stop there.