PROFESSOR: Hi. Welcome back to recitation. Today we're going to do another example of a definite integral. This time, it's going to be one that you're going to need to do some substitution for. You're not going to know the antiderivative right off the top of your head, I think.

So, I've got it on the board behind me. So it's compute the integral from minus 2 to 2 of the function x squared cosine of the quantity x cubed over 8 dx. So, this is a kind of a weird integrand. Roughly, you know, very, very rough graph of kind of what it looks like. So this is sort of the area of this region. It's a positive function over this interval. So it's the area of this region that I'm asking you to compute. So, all right, so definite integral. Why don't you pause the video, take a couple minutes, work this out yourself, come back, we can work it out together.

All right, welcome back. So, from looking at this integrand here, it seems to me that I don't know immediately off the top of my head what its antiderivative is. So, I can't just sort of apply the fundamental theorem of calculus directly without a little bit of work.

So there are two different ways that I can go about computing this then, about computing this definite integral. One is, that I can forget for the minute that it's a definite integral and compute the antiderivative and then use the fundamental theorem of calculus. So, let's try that way first and then we'll do it a second way as well. So, method one is to compute the antiderivative. So we compute the antiderivative of x squared cosine of x cubed over 8 dx. So here, it's not a definite integral anymore. Now I'm just looking at the antiderivative. And so, OK, so either by intelligent guessing or that-- I forget if that's what Professor Jerison called it-- or by just a substitution method. So, a good thing to substitute here, is we see inside this function we have this x cubed part and then outside we have an x squared part. So x squared dx, that's closely related to the differential of x cubed.

So we might try the substitution-- we might try the substitution u equals-- well we can maybe get rid of all of that at once-- so make u equals x cubed over 8. So in that case, du is equal to 3/8 x squared dx. And so this integral is equal to the integral of-- well so, x squared dx, we have that. So we need to multiply and divide by 3/8. So this is, 8/3 cosine of u du.

All right, and so now, OK, so the 8/3, that's just a constant. That pulls out in front. That's not hard. So then the antiderivative of cosine u du, that's something we know. So this is 8/3 of minus sine-- sorry not minus; I was doing the derivative instead of the antiderivative. Just 8/3

sine u. OK. And good. So that's the anti-- plus the constant, which we don't need because we're going to plug this in the fundamental theorem of calculus in a minute.

So it's 8/3 sine u. And, OK, so, but that's not good. I need it back in terms of my x's. And so I have to back-substitute. So this is 8/3 sine of x cubed over 8.

All right, now I've got an antiderivative of this expression. So now I can take this antiderivative and apply fundamental theorem of calculus, Right? So, the integral from minus 2 to 2 of x squared cosine of x cubed over 8 dx is equal to-- well it's equal to this antiderivative-- 8/3 sine x cubed over 8 taken between x equals minus 2 and x equals 2.

So this is equal to 8/3 times sine of 1 minus 8/3 times sine of minus 1. OK, and sine of minus x is equal to minus sine x. So we could rewrite this as 16/3 times sine of 1. So that's the answer. That's the value of this integral. And we did this first method by first computing the antiderivative and then using it with the fundamental theorem of calculus.

The second way we can do it is the one that we just learned more recently in lecture. Which is to use substitution directly in the definite integral. So let me come over here and do that. So, method two. So now, when we do it this way, mostly this looks the same. So we start off, we don't go back to antiderivative. We start off with the definite integral. So that's one difference. We can use the exact same substitution. So we can use exactly the substitution that we used before, which was u equals x cubed over 8. That doesn't change. And we still have the same du is equal to 3 x squared over 8, but what does change-- sorry, dx. Good. You need a differential always to be equal to a differential.

So what does change is that this-- I need to change the bounds. Right? If I'm keeping it as definite integrals all the time, so I'm changing here from an integral with respect to x to an integral with respect to u. And so that means that when I write down the second integral with respect to u, the bounds that I write down have to be the bounds for u. Not for x.

So when x is equal to minus 2, I need to know what the corresponding value of u is. So in this case, when x is equal to minus 2, we see that the lower bound here-- when x is equal to minus 2-- the lower bound on u becomes u equals, well, minus 2 cubed over 8. That's minus 1. And when x is equal to 2, the upper bound, u becomes 2 cubed over 8. So u becomes 1. So we have this extra step when we do it this way, of changing the bounds.

OK, so I do that. So now I get the integral. So I'm going to write u equals minus 1 here just to

remind myself that I made this change to 1. OK, and now the inside transforms exactly the same. So it's 8/3 times cosine u du. OK, and now again, this is something for which I already know the antiderivative. So I apply the fundamental theorem of calculus here. So I get the integral is equal to 8/3 sine of u, where u goes between minus 1 and 1.

So the other thing that's changed here is that now I don't have to change this back in terms of x's. I have the values that I'm going to plug in here already in terms of u. So I don't have to switch back in terms of x's. And so I just plug these values in and take the difference. So this is equal to 8/3 sine of 1 minus 8/3 sine of minus 1, which as we said before is 16/3 times sine of 1.

OK, so we have these two different methods. They're very, very, very, similar in how you apply them. The key differences are, that when you do it, the substitution in terms of definite integrals, you have to change your bounds of integration. And what you get for changing your bounds of integration is at the end you don't have to switch back from u's into x's. Whereas, if you do it by first computing the antiderivative, well, you don't have any bounds of integration when you compute the antiderivative so you don't have to change them. But once you computed the antiderivative, then you have to go and make your back substitution. Put everything back in terms of x's before going on to compute the definite integral that you started with.

So the those are the two different ways you can do substitution in definite integrals and I'll end with that.