1. Compute the following derivatives:

   (a) $f'(x)$ where $f(x) = x^3 e^x$

   (b) $f^{(7)}(x)$, the seventh derivative of f, where $f(x) = \sin(2x)$

2. (a) Find the tangent line to $y = 3x^2 - 5x + 2$ at $x = 2$. Express your answer in the form $y = mx + b$ with slope m and y-intercept b.

(b) Show that the curve defined implicitly by the equation

$$xy^3 + x^3y = 4$$

has no horizontal tangent.

3. (a) Use the DEFINITION of the derivative to compute $\frac{d}{dx}\left(\frac{x}{x+1}\right)$.

(b) Compute the following limit:

$$\lim_{x \to \sqrt{3}} \frac{\tan^{-1}(x) - \pi/3}{x - \sqrt{3}}$$

where as usual $\tan^{-1}(x)$ denotes the inverse tangent function.

4. Sketch the graph of the function

$$y = \frac{x}{x^2 + 1}.$$

Be sure to identify in writing all local maxs and mins, regions where the function is increasing/decreasing, points of inflection, symmetries, and vertical or horizontal asymptotes (if any of these behaviors occur).

5. A poster is to be designed with 50 in^2 of printed type, 4 inch margins on both the top and the bottom, and 2 inch margins on each side. Find the dimensions of the poster which minimize the amount of paper used. (Be sure to indicate why the answer you found is a minimum.)

6. A highway patrol plane is flying 1 mile above a long, straight road, with constant ground speed of 120 m.p.h. Using radar, the pilot detects a car whose distance from the plane is 1.5 miles and decreasing at a rate of 136 m.p.h. How fast is the car traveling along the highway? (Hint: You may give an exact answer, or use the fact that $\sqrt{5} \approx 2.2$.)

7. Evaluate the following limits:

(a)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{1+\frac{2i}{n}}\,\right] \frac{2}{n}$$

(b)

$$\lim_{h\to 0} \frac{1}{h} \int_{2}^{2+h} \sin(x^2)\, dx$$

8. Compute the following definite integrals:

(a)

$$\int_0^{\pi/4} \tan x \sec^2 x \, dx$$

(b)

$$\int_1^2 x \ln x \, dx$$

9. Calculate the following indefinite integral:

$$\int \frac{x^2\,dx}{\sqrt{9 - x^2}}$$

10. The disk bounded by the circle $x^2 + y^2 = a^2$ is revolved about the y-axis to make a sphere. Then a hole of diameter a is bored through the sphere along the y-axis (from north to south pole, like a cored apple). Find the volume of the resulting "cored" sphere. (Hint: Draw a picture of the two-dimensional region to be revolved, and label parts of your picture to help set up the integration.)

11. The following integral has no elementary antiderivative:

$$\int_1^5 \frac{e^x}{x}\,dx$$

Use the trapezoid rule with two trapezoids and the following table of values (accurate to one decimal place) to estimate this definite integral.

x	1	2	3	4	5
e^x/x	2.7	3.7	6.7	13.6	29.7

(Hint: It may help to draw a very rough picture of the area under the curve you are computing, divided into two trapezoids.)

12. The rate of radioactive decay of a mass of Radium-226 (call it dm/dt) is proportional to the amount m of Radium present at time t. Suppose we begin with 100 milligrams of Radium at time $t = 0$.

   (a) Given that the half life of Radium-226 is roughly 1600 years (half life is the time it takes for the mass to decay by half), find a formula for the mass of Radium that remains after t years by solving a differential equation.

   (b) Find the amount of Radium remaining after 1000 years. Simplify your answer using the fact that $2^{-10/16} \approx .65$

13. Cornu's spiral is defined by the parametric equations

$$\begin{aligned} x = C(t) &= \int_0^t \cos(\pi u^2/2) du \\ y = S(t) &= \int_0^t \sin(\pi u^2/2) du \end{aligned}$$

That is, the parametric equations are given by Fresnel functions we met earlier in the semester.

Find the arc length of the spiral from $t = 0$ to a fixed time $t = t_0$.

14. (a) Find the Taylor series of $\ln(1+x)$ centered at $a = 0$.

(b) Determine the radius of convergence of this Taylor series.

(c) Use the first two non-zero terms of the power series you found in (a) to approximate $\ln 3/2$.

(d) Give an upper bound on the error in your approximation in (c) using Taylor's inequality.

15. (BONUS – Only attempt this problem if you are finished with the exam and have time to spare.)

    Prove or disprove the following statement:

$$\frac{x}{1+x^2} < \tan^{-1}(x) < x \qquad \text{for all } x > 0.$$