Derivatives of Sine and Cosine

Using the Creating the Derivative mathlet, select the (default) function $f(x) = \sin(x)$ from the pull-down menu in the lower left corner of the screen. Do not check any of the boxes.

Move the slider or use the >> button to display the graph of the sine function.

a) For approximately what values of x is the slope of $f(x) = \sin(x)$ equal to 0?

b) At approximately what values of x is the slope of $f(x) = \sin(x)$ largest?

c) For each of the values you listed in (b), is the slope positive or negative?

d) Use the information you have collected to sketch the graph of $f'(x)$, the derivative of the sine function.

e) Check the box next to the red $f'(x)$ to check your work.

Solution

a) For approximately what values of x is the slope of $f(x) = \sin(x)$ equal to 0?

The slope of the graph is 0 wherever the tangent line is horizontal. For the sine function, the tangent line is horizontal at the peaks and troughs of its graph: $x = ..., -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, ...$

b) At approximately what values of x is the slope of $f(x) = \sin(x)$ largest?

The slope is the largest when the graph is the steepest. For the sine function, the graph is steepest when crossing the x-axis, at $x = ... - 2\pi, -\pi, 0, \pi, 2\pi...$

c) For each of the values you listed in (b), is the slope positive or negative?

The slope of the graph of the sine function at the x-intercepts alternates between positive and negative as the graph goes up and down across the axis. The slope is positive at $x = ... - 2\pi, 0, 2\pi...$ and negative when $x = ... - \pi, \pi, 3\pi, ...$

d) Use the information you have collected to sketch the graph of $f'(x)$, the derivative of the sine function.

The graph you drew should have peaks at -2π, 0 and 2π and troughs at $-\pi$ and π. Its x-intercepts should occur at $x = ..., -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, ...$ Because the graph of $\sin(x)$ is smooth and continuous, your graph should be connected.

It turns out that the graph of the derivative of $\sin(x)$ is also smooth, but it's hard to deduce this just by looking at the graph.

e) Check the box next to the red $f'(x)$ to check your work.

Perhaps you already knew that the derivative of $\sin(x)$ is $\cos(x)$. Remember this exercise next time you have trouble remembering whether the derivative of $\cos(x)$ is positive or negative $\sin(x)$, whether the graph of $\sin(x)$ goes up first or down, or what the graph of $\cos(x)$ looks like.

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