

PROFESSOR: Welcome back to recitation. In this video segment we're going to look at the chain rule. And specifically we're going to answer a question that I've placed on the board.

I want you to find two values for θ so that $\frac{d}{d\theta}$ of the function $\cos^2 \theta$ to the fourth equals 0. At this point I should also point out a few things. One, θ is a variable in this case. And if you haven't seen it before, frequently we use it when we're dealing with trigonometric functions. θ often represents the variable that measures angle. So if you haven't seen θ before, this is what it looks like. That's how you say it. And I'm actually doing the same things we've done before. I'm taking the derivative with respect to the variable θ , of a function of θ .

The other thing I want to point out, which I think we know already, but just to be sure, is it that this squared here, $\cos^2 \theta$ to the fourth, means I'm taking cosine of θ to the fourth and I'm actually squaring it. So with that knowledge I would like us to use the chain rule to find two values for θ where this derivative is equal to 0.

So I'll give you a moment to take a stab at finding the derivative, setting it equal to 0 and finding some values for θ that make this equation, the derivative equal 0. And we'll come back and then I'll work it out as well.

OK, so the first thing we obviously need to do is be able to take the derivative of this function on the left-hand side. And I should say it's a little more complicated than an example you saw in the lecture, because in the lecture you were given an example with just two functions. So I want to write it a little differently just to show very obviously what the three functions are we're composing.

So this function I can rewrite it as $\cos^2 \theta$ to the fourth-- does that look like a 4, there we go-- and then I square that whole thing. That's really what the function is. OK? So we can see what is the outermost function here? The outermost function is actually the quantity of something squared. So the outermost function is x^2 . What's the next function in, in this composition? The next function in is the cosine function. And then the last function in is the function taking something, raising it to the fourth. So it's very important you understand sort of the composition, which is the outermost function, which is the innermost function? In order to do this chain rule.

Now as you saw in recitation, you were given the example dy -- you had y as a function of t -- sorry, not in recitation. In the lecture, you were given y as a function of t or a function of x and then you had to put one other variable in the middle. Here we're going to have a composition of three functions. So we need to have two other things sort of in the middle. So let's write this out. First, the outermost function, we'll call the whole thing y . So y is equal to x squared is the outermost function. So then this whole thing is x now. So then we'll write x is equal to cosine of w . And then w is equal to θ to the fourth. OK.

Again this is what you saw before in the lecture. So you write the outermost function. And then that function is a function of cosine of another one, cosine of w , and w is a function of θ to the fourth. So if I put all these back together, I have θ to the fourth in here, and then I square that, and I come back to the function I wanted.

Now we know from the lecture what we need to do to find, essentially, $dy/d\theta$. That's what we're looking for. So $dy/d\theta$, you'll see, you remember from the lecture, should be dy/dx times dx/dw times $dw/d\theta$. So it's slightly more complicated than we saw before because there's one more term. OK.

So now let's work out what these are. Well dy/dx is fairly straightforward. dy/dx is just $2x$. And then dx/dw , well what's the derivative of the cosine function? The derivative of cosine is negative sine. So this is times negative sine of w times-- and what's $dw/d\theta$? So w is the function θ to the fourth. So $dw/d\theta$ is 4θ to the third.

Now when you look at this you should remember, x we sort of inserted into the problem to make the problem easier, and w we inserted into the problem to make the problem easier for us to follow. So we don't want all these x 's and w 's. We want everything in terms of, in terms of θ . But what is w ? Well w is θ to the fourth. So I can replace that by θ to the fourth. And what is x ? x is cosine w . But w is θ to the fourth. So x is actually cosine of θ to the fourth. OK?

So I get $2 \cos^2 \theta$ times negative sine-- w again is θ to the fourth-- of θ to the fourth times 4θ to the third. Let's make this nicer. I'll bring the coefficient and the θ to the third in front and this minus sign in front. I get negative 8θ to the third cosine of θ to fourth sine of θ to the fourth.

OK, and then the problem asks to find where the derivative is equal to 0. Find two values. Now why did I ask you to find two values? Because one is very easy. If this thing is set equal to 0,

one value should stand out right away for theta that makes this product 0. And that is theta equals 0. So theta equals 0 is our easiest answer.

So if you didn't do that, then you wanted the more challenging stuff, you could have done the other things also. So what about cosine theta to the fourth? If we want a product of three things to be 0, then at least one of them has to be 0. So I could have set cosine theta to the fourth equal to 0. And for what values of-- an angle I should just say-- for what values of theta to the fourth is this going to be 0? Well this means that theta to the fourth is equal to either maybe pi over 2, or you could add another pi. OK? So we could say, well let's just say that's one example. Theta to the fourth equals pi over 2 would work right? Because cosine of pi over 2 is equal to 0. OK? And so then we could have said theta is equal to pi over 2 to the 1/4. That's another example there.

We could have also done, we could have added pi to this and gotten another answer. But I only asked for two. So I guess that I'm allowed to stop there.