How Tall Can a Tree Grow?

Tree height	h
Trunk diameter	d
Wood weight per unit volume	G
Volume of a cylinder	$C_1 d^2 h$
Volume of the tree trunk	$W = GC_3 d^2 h$
Volume of the tree trunk and crown	$W = GC_3 d^2 h$
Tree trunk loaded by its own weight *C_1, C_2, And C_3 are constants	$W = GC_3 d^2 h$ Equation #1

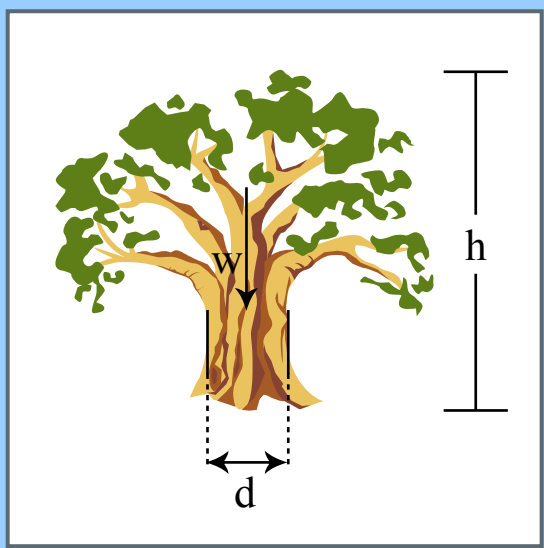
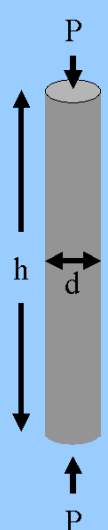


Figure by MIT OCW.

Column Buckling: At the Boston Nature Center you can learn by doing. A demonstration shows how different shape cylinders buckle.

Compress the column in the demonstration. Initially, the column stays straight. At some critical load, the column bows out, or "buckles."



$$d = C_5 h^{3/2}$$ Equation #2

E: is the stiffness of the material (how much it deforms for a given load) and is called "Young's Modulus."

C_4 is a constant

The demonstration has columns of different diameters (d) and different heights (h).

Check the formula by measuring the critical buckling load for several different columns. If you double the diameter, the buckling load increases by a factor of 2^4 or 16. If you double the height of the column, the buckling load decreases by a factor of 2^2 =4.

Tree height is controlled by column buckling.

As the tree gets taller, its weight increases.

At some point the trunk will buckle under its own weight. This happens when the weight (equation #1) equals the buckling load (equation #2)

$$d = C_5 h^{3/2}$$

Rearranging:

$$d = C_5 h^{3/2}$$

For different types of trees, G/E is a constant. (See page on wood structure and properties.)

Combining

$$d = C_5 h^{3/2}$$

We have

$$d = C_5 h^{3/2}$$

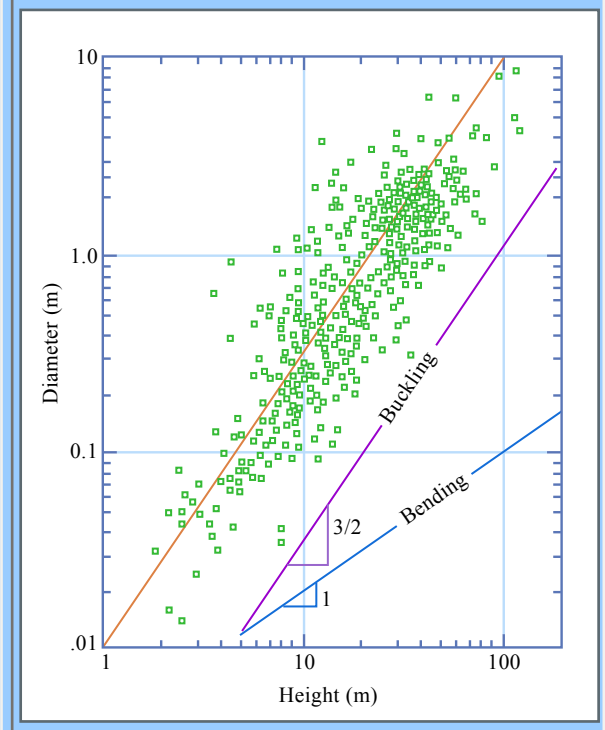
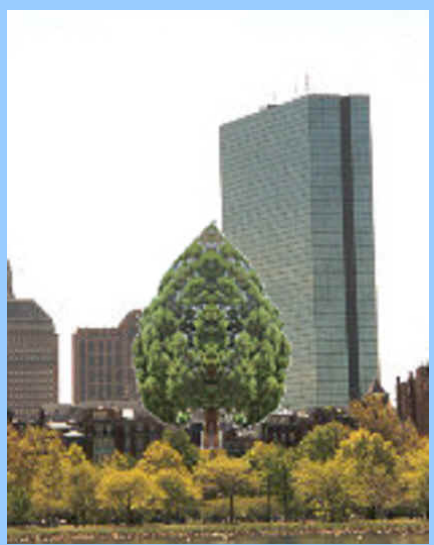


Figure by MIT OCW. After Bonner and McMahon (1983).

On a log-log plot, $d=C_5 h^{(3/2)}$ is a straight line with a slope of 3/2.

From Bonner, J.T. and McMahon, T. A. (1983). On Size and Life. Scientific American Books.

The American Forestry Association records the diameters and heights of the tallest individual trees of different types in the National Register of Big Trees. Each point on the plot (see above) corresponds to one individual record tree of a particular type.



For record trees:

$$d = C_5 h^{3/2}$$

Their height is limited by column buckling.

What is the tallest tree in the world?

The Mendocino Tree is a 367.5 feet high redwood and is 600-800 years old. It is in the remote Montgomery Woods State Reserve in California. This tree would be almost half the height of the Hancock Tower in Boston.