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3.21 Kinetics of Materials—Spring 2006

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Lecture 8: Solutions to the Diffusion Equation—II.

References

1. Balluffi, Allen, and Carter, *Kinetics of Materials*, Sections 5.1–5.2.4.

Key Concepts

- "Steady-state" diffusion is by definition time-independent. In some diffusion problems, the steady state is approached after diffusion has taken place for sufficiently long times. Solutions to the diffusion equation in the steady state are generally obtained by integration over the spatial variables, and boundary conditions are used to determine integration constants.

- Standard methods of obtaining "closed-form" time-dependent solutions to the diffusion equation for constant diffusivity include superposition of point sources, separation of variables, and Laplace transforms.

- The general form for the diffusion from a point source in a system diffusion in d dimensions is $c(\vec{r},t) = \frac{n_d}{(4\pi Dt)^{d/2}}\, e^{-\vec{r}\cdot\vec{r}}/(4Dt)$ (see *KoM* Table 5.1).

- The "thin-film solution" to the diffusion equation is a simple adaptation of the point-source solution in one dimension that describes the diffusion from a thin source of material at the surface of a semi-infinite body (see *KoM* Eq. 5.18).

- A distribution of point sources can be used to derive the error-function solution for one-dimensional interdiffusion in two semi-infinite bodies (see *KoM* Section 5.2.2). This serves as a prototype for solutions to more complex problems where the initial source has a finite geometry.

- Superposition of "image" sources provides a convenient method to solve some problems in semi-infinite systems where there is a zero-flux plane at the surface. A thin-film that is located at a finite depth below a surface (e.g., a dopant introduced by ion implantation) is a good example.

- Separation of variables is very useful technique for solving diffusion problems in finite geometries. In this method, solutions are sought that can be expressed as *products* of functions in a single variable: e.g., for one-dimensional diffusion, solutions of the form $c(x,t) = X(x)T(t)$. Substitution of this relation into the diffusion equation yields separate differential equations for the functions $X(x)$ and $T(t)$, and in generally this method leads to a solution that is an infinite series. Under favorable conditions (e.g., long diffusion times), only one or just a few of the terms in the series provide a good approximation of the exact solution.

Related Exercises in *Kinetics of Materials*

Review Exercises 5.1–5.11, pp. 114–126.