In problem 2 of P.S. 5, we define the relevant ensemble as an ensemble in which T,V,N and H (magnetic field) must be held constant. In the solutions, it was determined that the characteristic potential for this ensemble is given by:

$$\varphi = E - TS - HM$$

From the General Structure of Statistical Mechanics, we have:

$$-\beta\varphi = \ln\Gamma$$

Γ is the partition function of this ensemble and is given by:

$$\Gamma = \sum_{states} e^{-\beta(E_{state} - M_{state}H)}$$

Since the energy of the system is independent of the spin alignment of the individual particles, it is possible to set the energy as a constant and the partition function is just given by:

Now, the *total* magnetization of the system is given by

$$M = \sum_{i=1}^{N} ni\mu_0$$

The *microstates* available to each particle are either $ni = +1$ or $ni = -1$. The partition function can therefore be expressed as:

$$\Gamma = \sum_{ni=-1,+1} e^{\beta {}_{1}^{N} ni\mu_0 H}$$

The sum within the exponential is over the total number of particles of the system. From basic math, $e^{a+b} = e^a \cdot e^b$ and

$$\Gamma = \sum_{ni=-1,+1} e^{\beta {}_{1}^{N} ni\mu_0 H} = \sum_{ni=-1,+1} \prod_{i=1}^{N} e^{\beta ni\mu_0 H} = \prod_{i=1}^{N} \sum_{ni=-1,+1} e^{\beta ni\mu_0 H}$$

Since the particles are identical,

$$\Gamma = \prod_{i=1}^{N} \sum_{ni=-1,+1} e^{\beta ni\mu_0 H} = \left(\sum_{ni=-1,+1} e^{\beta ni\mu_0 H} \right)^N = \left(e^{\beta\mu_0 H} + e^{-\beta\mu_0 H} \right)^N$$

This could have bee done in less steps if we consider that, for a system of non-interacting distinguishable particles,

$$Q = q^N$$

We just needed to calculate the partition function q of a single particle. In this case, each particle can be in just two possible states, corresponding with a parallel or anti-parallel alignment with the field, and

$$q = e^{\beta\mu_0 H} + e^{-\beta\mu_0 H}$$