3.185 - Recitation Notes

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Topics Covered

- Drag Force
- Turbulence
- Forced Convection

Drag force can generally be written as (for both laminar and turbulent flow)

$$F_d = \int k f_x dA = k f_L A \text{ where } k = \frac{1}{2}\rho u^2$$

If flow is laminar, the drag force can be derived from τ, which is given by the Navier-Stoke's equation.

Friction Factor for Certain Geometries

Flat Plate

Laminar flow ($\text{Re} < 10^5$)

Boundary layer thickness: $\delta = \frac{5.0x}{\sqrt{\text{Re}_x}} = 5.0\sqrt{\frac{\nu x}{u_\infty}}$

Friction factor:
$$f_x = \frac{0.664}{\sqrt{\text{Re}_x}}$$
$$f_L = \frac{1.328}{\sqrt{\text{Re}_L}}$$

Drag force: $F_d = 0.664W\sqrt{\rho\mu u_\infty^3 L}$

Transition flow ($10^5 < \text{Re} < 10^7$)

Boundary layer thickness: $\delta = \frac{0.37x}{\text{Re}_x^{0.2}}$

Turbulent flow ($10^7 < \text{Re}$)

Friction factor:
$$f_L = \frac{0.455}{(\log \text{Re}_L)^{2.58}}$$
$$f_L = \frac{0.146}{\text{Re}_L^{0.2}}$$
(either one)

When flow is fully developed (i.e. x > entrance length), $f_x = f_L$.

Entrance length: $Le = \frac{u_{av}}{\nu}\frac{H^2}{100}$ where H: distance between plates

Sphere

Friction factor: $f = \frac{24}{\text{Re}}$ for Stoke's flow (Re < 0.1)

$f = 0.44$ for Re >> 1 (see fig. 12.4 in textbook)

Drag force: $F_D = 6\pi\mu R u_\infty$

Tubes

Friction factor: $f = \frac{16}{\text{Re}}$ for laminar flow, see eqn. (14-12) – (14-15) in textbook for friction factor on turbulent flow and various pipe roughness

Circular disc and square plates

Friction factor: $f = 1$ for Re >> 1 (see fig. 12.4 in textbook)

Forced Convection

Small Pr (Pr < 0.1)

- $\delta_c \; or \; \delta_T >> \delta_u$
- Weakly coupled: heat transfer does not affect fluid flow
- For heat transfer, u_x is assumed constant
- Solution similar to boundary layer problem in moving solid

Boundary layer thickness: $\delta_T = 3.6\sqrt{\frac{\alpha x}{u_\infty}}$

Boundary layer thickness ratio: $\frac{\delta_T}{\delta_u} = 0.72\,\text{Pr}^{-\frac{1}{2}}$

Temperature profile: $\frac{T - T_{env}}{T_\infty - T_{env}} = erf\left(\frac{y}{2\sqrt{\frac{\alpha x}{u_\infty}}}\right)$

$$q = h_x (T_s - T_\infty) \qquad J = h_D (C_s - C_\infty)$$

Heat/Mass flux: $h_x = \sqrt{\frac{k\rho C_P u_\infty}{\pi x}} \qquad h_{D,x} = \sqrt{\frac{D u_\infty}{\pi x}}$

$$h_L = 2\sqrt{\frac{k\rho C_P u_\infty}{\pi L}} \qquad h_{D,L} = 2\sqrt{\frac{D u_\infty}{\pi L}}$$

High Pr (Pr > 5)

Boundary layer thickness ratio: $\frac{\delta_T}{\delta_u}$ or $\frac{\delta_C}{\delta_u} = 0.975\,\mathrm{Pr}^{-\frac{1}{3}}$

Heat transfer coefficient *h* for forced convection can be found by computing the Nusselt number.

Nusselt Number: $h_x = \frac{Nu_x k}{x} \quad h_L = \frac{Nu_L k}{L}$

$$\left.\begin{aligned} Nu_x &= \frac{1}{\sqrt{\pi}} \mathrm{Re}_x^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{2}} \\ Nu_L &= \frac{2}{\sqrt{\pi}} \mathrm{Re}_L^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{2}} \end{aligned}\right\} \text{for small Pr (Pr < 0.1)}$$

$$\left.\begin{aligned} Nu_x &= \frac{0.564\,\mathrm{Re}_x^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{2}}}{1+0.90\sqrt{\mathrm{Pr}}} \\ Nu_L &= \frac{1.128\,\mathrm{Re}_L^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{2}}}{1+0.90\sqrt{\mathrm{Pr}}} \end{aligned}\right\} \text{for medium Pr}$$

$$\left.\begin{aligned} Nu_x &= 0.332\,\mathrm{Re}_x^{\frac{1}{2}} \mathrm{Pr}^{0.343} \\ Nu_L &= 0.664\,\mathrm{Re}_L^{\frac{1}{2}} \mathrm{Pr}^{0.343} \end{aligned}\right\} \text{for large Pr (Pr > 0.6)}$$

* also see class handout for others *Nu* relations