# 3.15 Electrical, Optical, and Magnetic Materials and Devices <br> Caroline A. Ross <br> Fall Term, 2005 

## Exam 1 (4 pages)

Closed book exam. Formulae and data are on the last 2.5 pages of the exam.
This takes 80 min and there are 80 points total. Be brief in your answers and use sketches.
Assume everything is at 300 K unless otherwise noted.

## 1. [20 points]

a) Draw sketches showing $g_{c}(E), g_{v}(E), f(E)$ and the carrier distributions for a semiconductor that is (i) intrinsic, (ii) n-type. (make the sketches with the E axis vertical.) [6]
b) Germanium has the same crystal structure as silicon but its band gap is 0.67 eV .If the total density of states in the conduction band $\left(\mathrm{N}_{\mathrm{c}}\right)$ and in the valence band $\left(\mathrm{N}_{\mathrm{v}}\right)$ are the same as they are for silicon, what value of $\mathrm{n}_{\mathrm{i}}$ would you expect for Ge at 300 K ? [6]
c) The Ge is now doped with B and with P . Both dopants have the same concentration. Assume the $B$ and $P$ energy levels are each 40 meV from the band edge. If $m_{n} * / m_{p} *=0.01$, draw the band diagram of the doped Ge as accurately as you can, showing $\mathrm{E}_{\mathrm{g}}, \mathrm{E}_{\mathrm{f}}$ and $\mathrm{E}_{\mathrm{i}}$. [6]
d) What electrical conductivity do you expect for the material in (c) compared to undoped Ge , and why? [2]

## 2. [30 points]

a) For a pn junction, draw the electron and hole concentrations vs distance outside the depletion region in the case of no bias, forward bias, and reverse bias, explaining briefly the shapes of the graphs. (2-3 sentences) [10]
b) Estimate the voltage you would need to apply to cause avalanche breakdown in a Si pn junction with $\mathrm{N}_{\mathrm{D}}$ in the n -side $=\mathrm{N}_{\mathrm{A}}$ in the p -side $=10^{15} \mathrm{~cm}^{-3}$. Assume that avalanche breakdown occurs at a field of $10^{5} \mathrm{~V} \mathrm{~cm}^{-1}$, and state any other assumptions you make. [20]

## 3. [30 points]

a) What factors affect the mobility of a carrier? (2-3 sentences) [6]
b) A piece of p-type Si with $\mathrm{N}_{\mathrm{A}}=10^{18} \mathrm{~cm}^{-3}$ and a length of 1 cm is heated at one end. This affects the value of $n_{i}$ as follows:


$$
\begin{array}{cc}
\text { Hot } & \text { Cold } \\
\mathrm{n}_{\mathrm{i}}=10^{12} \mathrm{~cm}^{-3} & \mathrm{n}_{\mathrm{i}}=10^{10} \mathrm{~cm}^{-3}
\end{array}
$$

Consider only the electrons in the Si , neglecting the motion of the holes. Where do drift and diffusion of the electrons occur? Estimate the electric field at the cold end of the Si . [16]
c) A BJT can be used to detect light by allowing the light to fall on the base region. How could you bias the two junctions in the BJT to get a good response to light? For your biasing scheme, draw the band structure and explain where the current(s) flow. [8]

## Properties

Si GaAs
$\mathrm{SiO}_{2}$
Ge

Atoms $/ \mathrm{cm}^{3}$, molecules $/ \mathrm{cm}^{3} \times 10^{22}$
Structure
Lattice constant (nm)
Density ( $\mathrm{g} / \mathrm{cm}^{3}$ )
Relative dielectric constant, $\varepsilon_{\mathrm{r}}$
Permittivity, $\varepsilon=\varepsilon_{\mathrm{r}} \varepsilon_{0}(\mathrm{farad} / \mathrm{cm}) \times 10^{-12}$
Expansion coefficient (dL/LdT) $\times\left(10^{-6} \mathrm{~K}\right)$
Specific Heat (joule/g K)
Thermal conductivity (watt/cm K)
Thermal diffusivity ( $\mathrm{cm}^{2} / \mathrm{sec}$ )
Energy Gap (eV)
Drift mobility ( $\mathrm{cm}^{2} /$ volt-sec)
Electrons
Holes
Effective density of states
$\left(\mathrm{cm}^{-3}\right) \times 10^{19}$
Conduction band
Valence band
Intrinsic carrier concentration $\left(\mathrm{cm}^{-3}\right)$


Properties of $\mathrm{Si}, \mathrm{GaAs}, \mathrm{SiO}_{2}$, and Ge at 300 K

## PHYSICAL CONSTANTS, CONVERSIONS, AND USEFUL COMBINATIONS

## Physical Constants

Avogadro constant
Boltzmann constant
Elementary charge
Planck constant

Speed of light
Permittivity (free space)
Electron mass
Coulomb constant
Atomic mass unit

## Useful Combinations

Thermal energy ( 300 K )
Photon energy
Coulomb constant
Permittivity (Si)
Permittivity (free space)
$\mathrm{N}_{\mathrm{A}}=6.022 \times 10^{23} \mathrm{particles} / \mathrm{mole}$
$k=8.617 \times 10^{-5} \mathrm{eV} / \mathrm{K}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
$e=1.602 \times 10^{-19} \mathrm{coulomb}$
$h=4.136 \times 10^{-15} \mathrm{eV} \cdot \mathrm{s}$
$=6.626 \times 10^{-34} \mathrm{joule} \cdot \mathrm{s}$
$c=2.998 \times 10^{10} \mathrm{em} / \mathrm{s}$
$\varepsilon_{0}=8.85 \times 10^{-14} \mathrm{farad} / \mathrm{cm}$
$m=9.1095 \times 10^{-31} \mathrm{~kg}$
$k_{\mathrm{c}}=8.988 \times 10^{9} \mathrm{newton}-\mathrm{m}^{2} /(\text { coulomb })^{2}$
$u=1.6606 \times 10^{-27} \mathrm{~kg}$
$k T=0.0258 \mathrm{eV} \simeq 1 \mathrm{eV} / 40$
$E=1.24 \mathrm{eV}$ at $\lambda=\mu \mathrm{m}$
$k_{\mathrm{c}} \mathrm{e}^{2} 1.44 \mathrm{eV} \cdot \mathrm{nm}$
$\varepsilon=\varepsilon_{\mathrm{r}} \varepsilon_{0}=1.05 \times 10^{-12} \mathrm{farad} / \mathrm{cm}$
$\varepsilon_{0}=55.3 \mathrm{e} / \mathrm{V} \cdot \mu \mathrm{m}$

## Prefixes

$\mathrm{k}=$ kilo $=10^{3} ; \mathrm{M}=$ mega $=10^{6} ; \mathrm{G}=$ giga $=10^{9} ; \mathrm{T}=$ tera $=10^{12}$
$\mathrm{m}=$ milli $=10^{-3} ; \mu=$ micro $=10^{-6} ; n=$ nano $=10^{-9} ; p=$ pica $=10^{-12}$

## Symbols for Units

Ampere (A), Coulomb (C), Farad (F), Gram (g), Joule (J), Kelvin (K)
Meter (m), Newton (N), Ohm ( $\Omega$ ), Second (s), Siemen (S), Tesla (T)
Volt (V), Watt (W), Weber (Wb)

## Conversions

$1 \mathrm{~nm}=10^{-9} \mathrm{~m}=10 \AA=10^{-7} \mathrm{~cm} ; 1 \mathrm{eV}=1.602 \times 10^{-9}$ Joule $=1.602 \times 10^{-12} \mathrm{erg} ;$
$1 \mathrm{eV} /$ particle $=23.06 \mathrm{kcal} / \mathrm{mol} ; 1$ newton $=0.102 \mathrm{~kg}_{\text {force }}$;
$10^{6}$ newton $/ \mathrm{m}^{2}=146 \mathrm{psi}=10^{7} \mathrm{dyn} / \mathrm{cm}^{2} ; 1 \mu \mathrm{~m}=10^{-4} \mathrm{~cm} 0.001 \mathrm{inch}=1 \mathrm{mil}=25.4 \mu \mathrm{~m}$; $1 \mathrm{bar}=10^{6} \mathrm{dyn} / \mathrm{cm}^{2}=10^{5} \mathrm{~N} / \mathrm{m}^{2} ; 1$ weber $/ \mathrm{m}^{2}=10^{4}$ gauss = 1 tesla;
1 pascal $=1 \mathrm{~N} / \mathrm{m}^{2}=7.5 \times 10^{-3}$ torr; $1 \mathrm{erg}=10^{-7}$ joule $=1 \mathrm{dyn}-\mathrm{cm}$

$$
\begin{aligned}
& \hline \mathrm{g}_{\mathrm{c}}(\mathrm{E}) \mathrm{dE}=\mathrm{m}_{\mathrm{n}} * \sqrt{ }\left\{2 \mathrm{~m}_{\mathrm{n}} *\left(\mathrm{E}-\mathrm{E}_{\mathrm{c}}\right)\right\} /\left(\pi^{2} \underline{h}^{3}\right) \quad(\underline{\mathrm{h}}=\mathrm{h}-\mathrm{bar}) \\
& \mathrm{g}_{\mathrm{v}}(\mathrm{E}) \mathrm{dE}=\mathrm{m}_{\mathrm{p}} * \sqrt{ }\left\{2 \mathrm{~m}_{\mathrm{p}} *\left(\mathrm{E}_{\mathrm{v}}-\mathrm{E}\right)\right\} /\left(\pi^{2} \underline{h}^{3}\right) \\
& \mathrm{f}(\mathrm{E})=1 /\left\{1+\exp \left(\mathrm{E}-\mathrm{E}_{\mathrm{f}}\right) / \mathrm{kT}\right\} \\
& \mathrm{n}=\mathrm{n}_{\mathrm{i}} \exp \left(\mathrm{E}_{\mathrm{f}}-\mathrm{E}_{\mathrm{i}}\right) / \mathrm{kT}, \quad \mathrm{p}=\mathrm{n}_{\mathrm{i}} \exp \left(\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{f}}\right) / \mathrm{kT} \\
& \mathrm{n}_{\mathrm{i}}=\mathrm{N}_{\mathrm{c}} \exp \left(\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{c}}\right) / \mathrm{kT} \quad \text { where } \mathrm{N}_{\mathrm{c}}=2\left\{2 \pi \mathrm{~m}_{\mathrm{n}} * \mathrm{kT} / \mathrm{h}^{2}\right\}^{3 / 2} \\
& \mathrm{np}=\mathrm{n}_{\mathrm{i}}^{2} \text { at equilibrium } \\
& \mathrm{n}_{\mathrm{i}}^{2}=\mathrm{N}_{\mathrm{c}} \mathrm{~N}_{\mathrm{v}} \exp \left(\mathrm{E}_{\mathrm{v}}-\mathrm{E}_{\mathrm{c}}\right) / \mathrm{kT}=\mathrm{N}_{\mathrm{c}} \mathrm{~N}_{\mathrm{v}} \exp \left(-\mathrm{E}_{\mathrm{g}}\right) / \mathrm{kT} \\
& \mathrm{E}_{\mathrm{i}}=\left(\mathrm{E}_{\mathrm{v}}+\mathrm{E}_{\mathrm{c}}\right) / 2+3 / 4 \mathrm{kT} \ln \left(\mathrm{~m}_{\mathrm{p}}^{* /} \mathrm{m}_{\mathrm{n}}^{*}\right) \\
& \\
& \mathrm{E}_{\mathrm{f}}-\mathrm{E}_{\mathrm{i}}=\mathrm{kT} \ln \left(\mathrm{n} / \mathrm{n}_{\mathrm{i}}\right)=-\mathrm{kT} \ln \left(\mathrm{p} / \mathrm{n}_{\mathrm{i}}\right) \\
& \quad \sim \mathrm{kT} \ln \left(\mathrm{~N}_{\mathrm{D}} / \mathrm{n}_{\mathrm{i}}\right) \text { ntype or }-\mathrm{kT} \ln \left(\mathrm{~N}_{\mathrm{A}} / \mathrm{n}_{\mathrm{i}}\right) \text { ptype }
\end{aligned}
$$

Drift: thermal velocity
drift velocity
Current density (electrons)
Current density (electrons \& holes)
Conductivity
Diffusion
Einstein relation:
R and G

$$
\mathrm{R}=\mathrm{G}=\mathrm{rnp}=\mathrm{r} \mathrm{n}_{\mathrm{i}}^{2} \text { at equilibrium }
$$

$$
\mathrm{dn} / \mathrm{dt}=\mathrm{dn} / \mathrm{dt}_{\mathrm{drift}}+\mathrm{dn} / \mathrm{dt}_{\text {diffn }}+\mathrm{dn} / \mathrm{dt}_{\text {thermal RG }}+\mathrm{dn} / \mathrm{dt}_{\text {ohher RG }}
$$

Fick's law $d n / d_{\text {diffn }}=1 / e \nabla J_{\text {diffn }}=D_{n} \mathrm{~d}^{2} \mathrm{n} / \mathrm{dx}^{2}$
so $\quad \mathrm{dn} / \mathrm{dt}=(1 / \mathrm{e}) \nabla\left\{\mathrm{J}_{\text {drift }}+\mathrm{J}_{\text {diffn }}\right\}+\mathrm{G}-\mathrm{R}$ $\mathrm{dn} / \mathrm{dt}_{\text {thermal }}=-\mathrm{n}_{\mathrm{l}} / \tau_{\mathrm{n}} \quad$ or $\quad \mathrm{dp} / \mathrm{dt}_{\text {thermal }}=-\mathrm{p}_{\mathrm{l}} / \tau_{\mathrm{p}}$
$\tau_{n}=1 / \mathrm{N}_{\mathrm{A}}$, or $\tau_{\mathrm{p}}=1 / \mathrm{r} \mathrm{N}_{\mathrm{D}} \quad \quad{ }_{\mathrm{n}}=\sqrt{ } \tau_{\mathrm{n}} \mathrm{D}_{\mathrm{n}}$, or $\quad \mathrm{p}=\sqrt{ } \tau_{\mathrm{p}} \mathrm{D}_{\mathrm{p}}$.
If traps dominate $\tau=1 / \mathrm{r}_{2} \mathrm{~N}_{\mathrm{T}}$ where $\mathrm{r}_{2} \gg \mathrm{r}$
pn junction

JFET

$$
\begin{aligned}
& \mathbf{E}=1 / \varepsilon_{0} \varepsilon_{\mathrm{r}} \int \rho(\mathrm{x}) \mathrm{dx} \text { where } \rho=\mathrm{e}\left(\mathrm{p}-\mathrm{n}+\mathrm{N}_{\mathrm{D}}-\mathrm{N}_{\mathrm{A}}\right) \\
& \mathbf{E}=-\mathrm{dV} / \mathrm{dx} \\
& e V_{o}=\left(E_{f}-E_{i}\right)_{\text {n-type }}-\left(E_{f}-E_{i}\right)_{\text {p-type }} \\
& =k T / e \ln \left(n_{n} / n_{p}\right) \text { or } k T / e \ln \left(N_{A} N_{D} / n_{i}^{2}\right) \\
& \mathbf{E}=\mathrm{N}_{\mathrm{A}} \mathrm{e} \mathrm{~d}_{\mathrm{p}} / \varepsilon_{0} \varepsilon_{\mathrm{r}}=\mathrm{N}_{\mathrm{D}} \mathrm{e} \mathrm{~d}_{\mathrm{p}} / \varepsilon_{\mathrm{o}} \varepsilon_{\mathrm{r}} \quad \text { at } \mathrm{X}=0 \\
& \mathrm{~V}_{\mathrm{o}}=\left(\mathrm{e} / 2 \varepsilon_{\mathrm{o}} \varepsilon_{\mathrm{r}}\right)\left(\mathrm{N}_{\mathrm{D}} \mathrm{~d}_{\mathrm{n}}{ }^{2}+\mathrm{N}_{\mathrm{A}} \mathrm{~d}_{\mathrm{p}}{ }^{2}\right) \\
& \mathrm{d}_{\mathrm{n}}=\sqrt{ }\left\{\left(2 \varepsilon_{0} \varepsilon_{\mathrm{r}} \mathrm{~V}_{\mathrm{o}} / \mathrm{e}\right)\left(\mathrm{N}_{\mathrm{A}} /\left(\mathrm{N}_{\mathrm{D}}\left(\mathrm{~N}_{\mathrm{D}}+\mathrm{N}_{\mathrm{A}}\right)\right)\right\}\right. \\
& \mathrm{d}=\mathrm{d}_{\mathrm{p}}+\mathrm{d}_{\mathrm{n}}=\sqrt{ }\left\{\left(2 \varepsilon_{\mathrm{o}} \varepsilon_{\mathrm{r}}\left(\mathrm{~V}_{\mathrm{o}}+\mathrm{V}_{\mathrm{A}}\right) / \mathrm{e}\right)\left(\mathrm{N}_{\mathrm{D}}+\mathrm{N}_{\mathrm{A}}\right) / \mathrm{N}_{\mathrm{A}} \mathrm{~N}_{\mathrm{D}}\right\} \\
& \mathrm{J}=\mathrm{J}_{\mathrm{o}}\left\{\operatorname{expeV} \mathrm{~V}_{\mathrm{A}} / \mathrm{kT}-1\right\} \text { where } \mathrm{J}_{\mathrm{o}}=\mathrm{en}_{\mathrm{i}}{ }^{2}\left\{\mathrm{D}_{\mathrm{p}} / \mathrm{N}_{\mathrm{D}} \mathrm{p}_{\mathrm{p}}+\mathrm{D}_{\mathrm{n}} / \mathrm{N}_{\mathrm{A}}{ }_{\mathrm{n}}\right\} \\
& \text { Transistor BJT gain } \beta=\mathrm{I}_{\mathrm{C}} / \mathrm{I}_{\mathrm{B}} \sim \mathrm{I}_{\mathrm{E}} / \mathrm{I}_{\mathrm{B}}=\mathrm{N}_{\mathrm{A}, \mathrm{E}} / \mathrm{N}_{\mathrm{D}, \mathrm{~B}} \\
& \mathrm{I}_{\mathrm{E}}=\left(\mathrm{eD}_{\mathrm{p}} / \mathrm{w}\right)\left(\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{N}_{\mathrm{D}, \mathrm{~B}}\right) \exp \left(\mathrm{e} \mathrm{~V}_{\mathrm{EB}} / \mathrm{kT}\right) \\
& \mathrm{V}_{\text {SD, sat }}=\left(\mathrm{eN}_{\mathrm{D}} \mathrm{t}^{2} / 8 \varepsilon_{0} \varepsilon_{\mathrm{r}}\right)-\left(\mathrm{V}_{\mathrm{o}}+\mathrm{V}_{\mathrm{G}}\right)
\end{aligned}
$$

