### 3.15 Electrical, Optical, and Magnetic Materials and Devices

## Caroline A. Ross

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## Exam 2 (6 pages)

Closed book exam. Formulae and data are on the last 4 pages of the exam.
This takes $\mathbf{8 0} \mathbf{~ m i n}$ and there are 80 points total. Be brief in your answers and use sketches. Assume everything is at 300 K unless otherwise noted.

1. [25 points] A JFET is constructed like this: the two gate regions are $n+$ and the rest of the material is p type.

a) Assume that the source is grounded (at zero volts). Draw a sketch of how the current $I_{D}$ flowing out of the drain varies with voltage $\mathrm{V}_{\mathrm{D}}$ when the gate is at zero volts. Consider both positive and negative values of $\mathrm{V}_{\mathrm{D}}$ and explain the shape of your graph. (3-4 sentences) [9]
b) Now draw another sketch showing again $I_{D}$ vs. $V_{D}$ but this time draw different graphs corresponding to different values of gate voltage $\mathrm{V}_{\mathrm{G}}$ (both positive and negative). Explain briefly how gate voltage affects the I-V plot. [10]
c) Give three reasons why a MOSFET is preferable to a JFET. [6]
2. [33 points] In one of the problem sets we considered two ways of making white light using an LED. Here we will consider a third method. In this new scheme, the device looks like this:

a) What is the purpose of the phosphor-coated cup in this device? Explain how you can get white light. [6]
b) What color LED would you use? Choose a possible material and substrate for the LED, explaining your choice. [7] (some of the data at the end of the exam may be helpful)
c) Draw a possible band structure for your LED, in the unbiased case, and explain how bias affects the band structure. [7]
d) Explain concisely the differences between the spectral output of an LED, such as the one you just drew, compared with a semiconductor laser. [6]
e) If your LED is only 0.1 mm long, how does this affect its output? [7]
3. [22 points] We are designing a photovoltaic system. The solar cell we have available produces an output as shown below: its internal resistance is 0.2 ohms.

a) Estimate the maximum power we can produce from the solar cell. [6]
b) What load resistance would you use to maximize the output power? [4]
c) Mention three methods to improve the efficiency of a solar cell. (1 sentence each). [6]
d) Solar cells can be made from amorphous silicon, and are commonly used in devices such as calculators. Explain the reason for using amorphous Si in place of crystal Si (3-4 sentences). [6]


Figure by MIT OCW.


Figure by MIT OCW.


Properties of $\mathrm{Si}, \mathrm{GaAs}, \mathrm{SiO}_{2}$, and Ge at 300 K
Figure by MIT OCW.

## Useful equations

$$
\begin{aligned}
& \mathrm{g}_{\mathrm{c}}(\mathrm{E}) \mathrm{dE}=\mathrm{m}_{\mathrm{n}}{ }^{*} \sqrt{ }\left\{2 \mathrm{~m}_{\mathrm{n}}{ }^{*}\left(\mathrm{E}-\mathrm{E}_{\mathrm{c}}\right)\right\} /\left(\pi^{2} \underline{\mathrm{~h}}^{3}\right) \quad(\underline{\mathrm{h}}=\mathrm{h} \text {-bar }) \\
& \mathrm{g}_{\mathrm{v}}(\mathrm{E}) \mathrm{dE}=\mathrm{m}_{\mathrm{p}} * \sqrt{ }\left\{2 \mathrm{~m}_{\mathrm{p}} *\left(\mathrm{E}_{\mathrm{v}}-\mathrm{E}\right)\right\} /\left(\pi^{2} \underline{\underline{h}}^{3}\right) \\
& \mathrm{f}(\mathrm{E})=1 /\left\{1+\exp \left(\mathrm{E}-\mathrm{E}_{\mathrm{f}}\right) / \mathrm{kT}\right\} \\
& \mathrm{n}=\mathrm{n}_{\mathrm{i}} \exp \left(\mathrm{E}_{\mathrm{f}}-\mathrm{E}_{\mathrm{i}}\right) / \mathrm{kT}, \quad \mathrm{p}=\mathrm{n}_{\mathrm{i}} \exp \left(\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{f}}\right) / \mathrm{kT} \\
& n_{i}=N_{c} \exp \left(E_{i}-E_{c}\right) / k T \quad \text { where } N_{c}=2\left\{2 \pi m_{n} * k T / h^{2}\right\}^{3 / 2} \\
& n \mathrm{n}=\mathrm{n}_{\mathrm{i}}^{2} \text { at equilibrium } \\
& n_{i}^{2}=N_{c} N_{v} \exp \left(E_{v}-E_{c}\right) / k T=N_{c} N_{v} \exp \left(-E_{g}\right) / k T \\
& \mathrm{E}_{\mathrm{i}}=\left(\mathrm{E}_{\mathrm{v}}+\mathrm{E}_{\mathrm{c}}\right) / 2+3 / 4 \mathrm{kT} \ln \left(\mathrm{~m}_{\mathrm{p}}{ }^{*} / \mathrm{m}_{\mathrm{n}}{ }^{*}\right) \\
& \mathrm{E}_{\mathrm{f}}-\mathrm{E}_{\mathrm{i}}=\mathrm{kT} \ln \left(\mathrm{n} / \mathrm{n}_{\mathrm{i}}\right)=-\mathrm{kT} \ln \left(\mathrm{p} / \mathrm{n}_{\mathrm{i}}\right) \\
& \sim \mathrm{kT} \ln \left(\mathrm{~N}_{\mathrm{D}} / \mathrm{n}_{\mathrm{i}}\right) \text { type or }-\mathrm{kT} \ln \left(\mathrm{~N}_{\mathrm{A}} / \mathrm{n}_{\mathrm{i}}\right) \text { type }
\end{aligned}
$$

Drift: thermal velocity

$$
1 / 2 \mathrm{mv}^{2}{ }_{\text {thermal }}^{2}=3 / 2 \mathrm{kT}
$$

drift velocity
Current density (electrons)
Current density (electrons \& holes)
Conductivity
Diffusion
Einstein relation:
R and G
Fisk's law $\quad d n / \mathrm{dt}_{\text {diff }}=1 / e \nabla J_{\text {diff }}=D_{n} d^{2} n / d x^{2}$
so $\quad \mathrm{dn} / \mathrm{dt}=(1 / \mathrm{e}) \nabla\left\{\mathrm{J}_{\text {drift }}+\mathrm{J}_{\text {diff }}\right\}+\mathrm{G}-\mathrm{R}$
$\mathrm{dn} / \mathrm{dt}_{\text {thermal }}=-\mathrm{n}_{1} / \tau_{\mathrm{n}} \quad$ or $\quad \mathrm{dp} / \mathrm{dt}_{\text {thermal }}=-\mathrm{p}_{\mathrm{l}} / \tau_{\mathrm{p}}$
$\tau_{\mathrm{n}}=1 / \mathrm{rN}_{\mathrm{A}}$, or $\tau_{\mathrm{p}}=1 / \mathrm{rN}_{\mathrm{D}}$

$$
\mathrm{L}_{\mathrm{n}}=\sqrt{ } \tau_{\mathrm{n}} \mathrm{D}_{\mathrm{n}}, \text { or } \mathrm{L}_{\mathrm{p}}=V_{\tau_{\mathrm{p}}} \mathrm{D}_{\mathrm{p}}
$$

If traps dominate $\tau=1 / \mathrm{r}_{2} \mathrm{~N}_{\mathrm{T}}$ where $\mathrm{r}_{2} \gg \mathrm{r}$
$\mathrm{J}=\mathrm{ne} \mathrm{v}_{\mathrm{d}}$
$\mathrm{J}=\mathrm{e}\left(\mathrm{n} \mu_{\mathrm{n}}+\mathrm{p} \mu_{\mathrm{h}}\right) \mathbf{E}$
$\sigma=\mathrm{J} / \mathbf{E}=\mathrm{e}\left(\mathrm{n} \mu_{\mathrm{n}}+\mathrm{p} \mu_{\mathrm{h}}\right)$
$\mathrm{J}=\mathrm{e}_{\mathrm{n}} \nabla \mathrm{n}+\mathrm{eD}_{\mathrm{p}} \nabla \mathrm{p}$
$\mathrm{D}_{\mathrm{n}} / \mu_{\mathrm{n}}=\mathrm{kT} / \mathrm{e}$
$\mathrm{R}=\mathrm{G}=\mathrm{rnp}=\mathrm{r} \mathrm{n}_{\mathrm{i}}^{2} \quad$ at equilibrium
$\mathrm{dn} / \mathrm{dt}=\mathrm{dn} / \mathrm{dt}_{\text {drift }}+\mathrm{dn} / \mathrm{dt}_{\text {diff }}+\mathrm{dn} / \mathrm{dt}_{\text {thermal RG }}+\mathrm{dn} / \mathrm{dt}_{\text {other RG }}$
pn junction

$$
\begin{aligned}
& \mathbf{E}=1 / \varepsilon_{0} \varepsilon_{\mathrm{r}} \int \rho(\mathrm{x}) \mathrm{dx} \quad \text { where } \rho=\mathrm{e}\left(\mathrm{p}-\mathrm{n}+\mathrm{N}_{\mathrm{D}}-\mathrm{N}_{\mathrm{A}}\right) \\
& \mathbf{E}=-\mathrm{dV} / \mathrm{dx} \\
& e V_{o}=\left(E_{f}-E_{i}\right)_{n-t y p e}-\left(E_{f}-E_{i}\right)_{p-t y p e} \\
& =\mathrm{kT} / \mathrm{e} \ln \left(\mathrm{n}_{\mathrm{n}} / \mathrm{n}_{\mathrm{p}}\right) \text { or } \mathrm{kT} / \mathrm{e} \ln \left(\mathrm{~N}_{\mathrm{A}} \mathrm{~N}_{\mathrm{D}} / \mathrm{n}_{\mathrm{i}}{ }^{2}\right) \\
& \mathbf{E}=\mathrm{N}_{\mathrm{A}} \mathrm{e} \mathrm{~d}_{\mathrm{p}} / \varepsilon_{0} \varepsilon_{\mathrm{r}}=\mathrm{N}_{\mathrm{D}} \mathrm{e} \mathrm{~d}_{\mathrm{p}} / \varepsilon_{0} \varepsilon_{\mathrm{r}} \quad \text { at } \mathrm{x}=0 \\
& \mathrm{~V}_{\mathrm{o}}=\left(\mathrm{e} / 2 \varepsilon_{0} \varepsilon_{\mathrm{r}}\right)\left(\mathrm{N}_{\mathrm{D}} \mathrm{~d}_{\mathrm{n}}{ }^{2}+\mathrm{N}_{\mathrm{A}} \mathrm{~d}_{\mathrm{p}}{ }^{2}\right) \\
& \mathrm{d}_{\mathrm{n}}=\sqrt{ }\left\{\left(2 \varepsilon_{\mathrm{o}} \varepsilon_{\mathrm{r}} \mathrm{~V}_{\mathrm{o}} / \mathrm{e}\right)\left(\mathrm{N}_{\mathrm{A}} /\left(\mathrm{N}_{\mathrm{D}}\left(\mathrm{~N}_{\mathrm{D}}+\mathrm{N}_{\mathrm{A}}\right)\right)\right\}\right. \\
& \mathrm{d}=\mathrm{d}_{\mathrm{p}}+\mathrm{d}_{\mathrm{n}}=\sqrt{ }\left\{\left(2 \varepsilon_{0} \varepsilon_{\mathrm{r}}\left(\mathrm{~V}_{\mathrm{o}}+\mathrm{V}_{\mathrm{A}}\right) / \mathrm{e}\right)\left(\mathrm{N}_{\mathrm{D}}+\mathrm{N}_{\mathrm{A}}\right) / \mathrm{N}_{\mathrm{A}} \mathrm{~N}_{\mathrm{D}}\right\} \\
& \mathrm{J}=\mathrm{J}_{\mathrm{o}}\left\{\exp \mathrm{eV}_{\mathrm{A}} / \mathrm{kT}-1\right\} \text { where } \mathrm{J}_{\mathrm{o}}=\mathrm{en}_{\mathrm{i}}^{2}\left\{\mathrm{D}_{\mathrm{p}} / \mathrm{N}_{\mathrm{D}} \mathrm{~L}_{\mathrm{p}}+\mathrm{D}_{\mathrm{n}} / \mathrm{N}_{\mathrm{A}} \mathrm{~L}_{\mathrm{n}}\right\} \\
& \text { Transistor BJT gain } \beta=\mathrm{I}_{\mathrm{C}} / \mathrm{I}_{\mathrm{B}} \sim \mathrm{I}_{\mathrm{E}} / \mathrm{I}_{\mathrm{B}}=\mathrm{N}_{\mathrm{A}, \mathrm{E}} / \mathrm{N}_{\mathrm{D}, \mathrm{~B}} \\
& \mathrm{I}_{\mathrm{E}}=\left(\mathrm{eD}_{\mathrm{p}} / \mathrm{w}\right)\left(\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{N}_{\mathrm{D}, \mathrm{~B}}\right) \exp \left(\mathrm{eV}_{\mathrm{EB}} / \mathrm{kT}\right) \\
& \text { JFET } \quad \mathrm{V}_{\mathrm{SD}, \text { sat }}=\left(\mathrm{eN}_{\mathrm{D}} \mathrm{t}^{2} / 8 \varepsilon_{\mathrm{o}} \varepsilon_{\mathrm{r}}\right)-\left(\mathrm{V}_{\mathrm{o}}+\mathrm{V}_{\mathrm{G}}\right)
\end{aligned}
$$

Photodiode and photovoltaic

$$
\begin{array}{lc}
\mathrm{I}=\mathrm{I}_{\mathrm{o}}+\mathrm{I}_{\mathrm{G}} & \mathrm{~V}=\mathrm{I}\left(\mathrm{R}_{\mathrm{PV}}+\mathrm{R}_{\mathrm{L}}\right) \\
\mathrm{I}=\mathrm{I}_{\mathrm{o}}\{\operatorname{expeV} / \mathrm{kT}-1\}+\mathrm{I}_{\mathrm{G}} & \text { Power }=\mathrm{IV}
\end{array}
$$

Wavelength

$$
\lambda(\mu \mathrm{m})=1.24 / \mathrm{E}_{\mathrm{g}}(\mathrm{eV})
$$

## Band structure

Effective mass: $\quad \mathrm{m}^{*}=\hbar^{2}\left(\partial^{2} E / \partial k^{2}\right)^{-1}$
Momentum of an electron typically $\pi / \mathrm{a} \sim 10^{10} \mathrm{~m}^{-1}$
Momentum of a photon $=2 \pi / \lambda \sim 10^{7} \mathrm{~m}^{-1}$
Uncertainly principle $\Delta x \Delta p \geq \hbar$
Lasers
probability of absorption $=B_{13}$, stimulated emission $=B_{31}$, spontaneous emission $=A_{31}$
$\mathrm{N}_{3}=\mathrm{N}_{1} \exp \left(-\mathrm{h} v_{31} / \mathrm{kT}\right)$
Planck $\rho(v) \mathrm{d} v=\left\{8 \pi \mathrm{~h} v^{3} / \mathrm{c}^{3}\right\} /\{\exp (\mathrm{h} v / \mathrm{kT})-1\} \mathrm{d} v$

$$
\mathrm{B}_{13}=\mathrm{B}_{31}
$$

and $\quad \mathrm{A}_{31} / \mathrm{B}_{31}=8 \pi \mathrm{~h} v^{3} / \mathrm{c}^{3} \quad$ (Einstein relations)
Cavity modes $\quad v=\mathrm{cN} / 2 \mathrm{~d}, \mathrm{~N}$ an integer.
Fibers
Attenuation $(\mathrm{dB}) \quad=\{10 / \mathrm{L}\} \log \left(\mathrm{P}_{\text {in }} / \mathrm{P}_{\text {out }}\right) \quad \mathrm{L}=$ fiber length
Snell's law: $\quad \mathrm{n} \sin \phi=\mathrm{n}$ ' $\sin \phi$ '
Dispersion coefft. $\mathrm{D}_{\lambda}=-\left\{\lambda_{o} / c\right\}\left(\partial^{2} n / \partial \lambda^{2}\right)_{\lambda=\lambda_{0}} \mathrm{ps} / \mathrm{km} . \mathrm{nm}$

$$
\sigma_{t}=\sigma_{\lambda} L D_{\lambda}
$$

## PHYSICAL CONSTANTS, CONVERSIONS, AND USEFUL COMBINATIONS

## Physical Constants

Avogadro constant
Boltzmann constant
Elementary charge
Planck constant

Speed of light
Permittivity (free space)
Electron mass
Coulomb constant
Atomic mass unit

## Useful Combinations

Thermal energy ( 300 K )
Photon energy
Coulomb constant
Permittivity (Si)
Permittivity (free space)
Prefixes
$\mathrm{k}=$ kilo $=10^{3} ; \mathrm{M}=$ mega $=10^{6} ; \mathrm{G}=$ giga $=10^{9} ; \mathrm{T}=$ tera $=10^{12}$
$\mathrm{m}=$ milli $=10^{-3} ; \mu=$ micro $=10^{-6} ; n=$ nano $=10^{-9} ; p=$ pica $=10^{-12}$

## Symbols for Units

Ampere (A), Coulomb (C), Farad (F), Gram (g), Joule (J), Kelvin (K)
Meter (m), Newton (N), Ohm ( $\Omega$ ), Second (s), Siemen (S), Tesla (T)
Volt (V), Watt (W), Weber (Wb)

## Conversions

$1 \mathrm{~nm}=10^{-9} \mathrm{~m}=10 \AA=10^{-7} \mathrm{~cm} ; 1 \mathrm{eV}=1.602 \times 10^{-9}$ Joule $=1.602 \times 10^{-12} \mathrm{erg} ;$
$1 \mathrm{eV} /$ particle $=23.06 \mathrm{kcal} / \mathrm{mol} ; 1$ newton $=0.102 \mathrm{~kg}_{\text {force }}$;
$10^{6}$ newton $/ \mathrm{m}^{2}=146 \mathrm{psi}=10^{7} \mathrm{dyn} / \mathrm{cm}^{2} ; 1 \mu \mathrm{~m}=10^{-4} \mathrm{~cm} 0.001 \mathrm{inch}=1 \mathrm{mil}=25.4 \mu \mathrm{~m}$;
$1 \mathrm{bar}=10^{6} \mathrm{dyn} / \mathrm{cm}^{2}=10^{5} \mathrm{~N} / \mathrm{m}^{2} ; 1$ weber $/ \mathrm{m}^{2}=10^{4}$ gauss = 1 tesla;
1 pascal $=1 \mathrm{~N} / \mathrm{m}^{2}=7.5 \times 10^{-3}$ torr; $1 \mathrm{erg}=10^{-7}$ joule $=1$ dyn -cm

