Lecture 18, Nature Sandwich Notes, 3.054

Sandwich structures in nature

- Previously, saw sand structures efficient in resisting bending, buckling
- Sandwich panels also appear in nature:
  - leaves of monocotyledon plants (grasses, corn, iris)
  - skulls (esp. birds)
  - shells of some arthropods (e.g. horseshoe crab)
  - cuttlefish bone (mollusk)

Leaves

- Leaves must provide for structural support as well as large surface area for photosynthesis
- Iris, cattail, ryegrass, giant feather grass - leaves all sandwich structures

Iris leaves

- Nearly fully dense ribs (sclerenchyma) running along length of outer surfaces
- Ribs separated by a core of foam-like parenchyma cells

Leaves

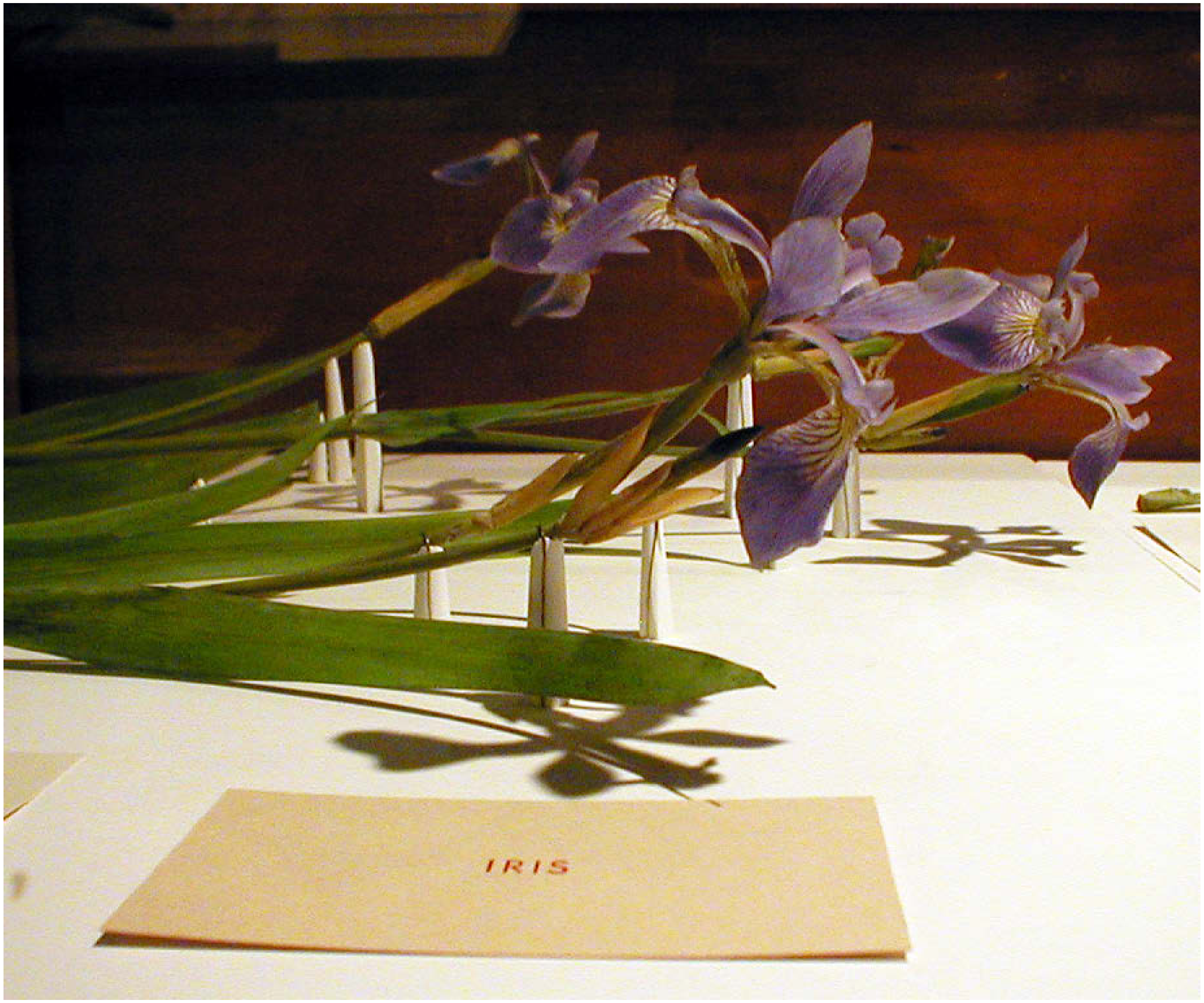
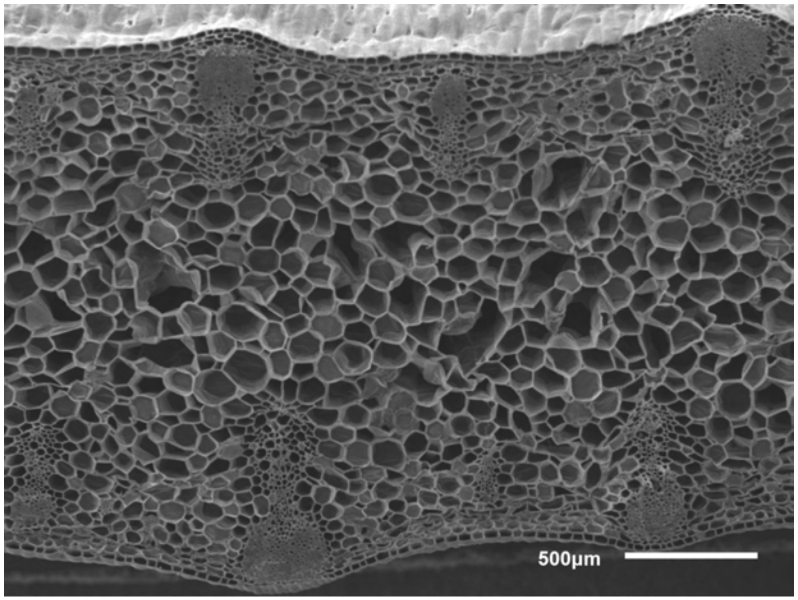
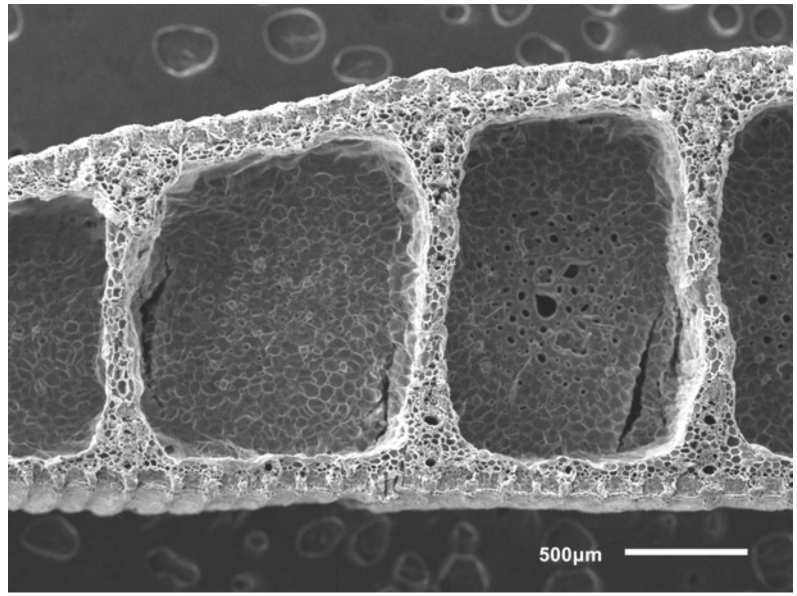


Photo of Blaschka glass flowers (iris) at the Harvard Museum of Natural History. Courtesy of Andrew Kuchling on Flickr. License: CC-BY.

Leaves



Iris



Cattail
(Bulrush)

Leaves

Figures removed due to copyright restrictions. Figure 1: Vincent, J. F. V. "The Mechanical Design of Grass." *Journal of Material Science* 17 (1982): 856–60. Figure 2: Vincent, J. F. V. "Strength and Fracture of Grasses." *Journal of Material Science* 26 (1991): 1947-50.

Iris leaf

Figures removed due to copyright restrictions. See Figures 3 and 4: Gibson, L. J., M. F. Ashby, et al. "Structure and Mechanics of the Iris Leaf." *Journal of Material Science* 23 (1988): 3041-48.

- Outer face – ribs connected by single layer of roughly square cells
  – jointly act as fiber reinforced composite
- Measurements of leaf microstructure summarized in Table
- Can analyze leaf as a sandwich structure
- Compare analysis with bending tests on fresh iris leaves
  – cantilevers with weights hung from free end ($B_1 = 3,\ B_2 = 1$)

$$\left(\frac{\delta}{P}\right)_{\text{calc}} = \frac{2\,l^3}{3E_f\,b\,t\,c^2} + \frac{l}{G_c^*\,b\,c}$$

t,c	measured from micrographs (Table)
b,l	from beam bending tests
E_f, G_c^*	need to estimate

E_f
- can be estimated from $E_{rib}V_{rib}$ in face (neglect contribution of square cells in face)
- ribs — sclerenchyma
- previous studies — sclerenchyma from grass leaf fibers E_{scler} = 2-23 GPa
- tensile tests on iris leaves E_{rib} = 21 GPa
- volume fraction of ribs in the faces is 0.39

  $E_f = 0.39E_{rib} = 8.2$ GPa

G_c^*
- assume tissue fresh (E parenchyma constant at high/normal turgor pressure)
- data for $E_{parenchyma}$ = 0.5-6 MPa
- take $E_{parenchyma} \approx 4$ MPa
- $G_c^* \sim 1/2\ E_{parenchyma} = 2$ MPa

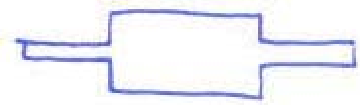
Parenchyma Properties

Plant material	Young's modulus, E^* or shear modulus, G^* (MPa)	Compressive strength, σ^*_{comp} (MPa)	Reference
Apple	$E^* = 0.31\text{-}3.46$	0.66	Oye et al., 2007
Apple	$E^* = 2.8\text{-}5.8$	0.25-0.37	Lin & Pitt, 1986
Apple	$G^* = 1\text{-}6$		Vincent, 1989
Potato	$E^* = 3.6$	1.3	Lin & Pitt, 1986
Potato	$E^* = 3.5$		Scanlon et al, 1996
Potato	$E^* = 5.5$	0.27	Hiller & Jeronimides, 1996
Potato	$G^* = 0.5\text{-}1$		Scanlon et al., 1996; 1998
Carrot	$E^* = 2\text{-}14$		Georget et al., 2003

Data for fresh, wet tissue, at normal turgor.

- Using sandwich beam theory, can estimate P/δ (Table)
- Calculation complicated by irregular thickness of core across section:

- Rough attempt to account for this by dividing cross-section into sub-units



- Found calculated P/ρ overestimated measured P/ρ by 16-83%
- Agreement OK for the various approximations and estimates mode

Strength of the leaf

Faces: $\sigma_{ys} =?$

- Previous tests on tensile strength of gross leaves found
  σ_f (in MPa) $= 1.44\ (\underset{\substack{\uparrow \\ \text{vol. fraction}}}{V_{\text{sclerenchyma}}} \times 100) + 1.53$
- In iris, ribs (assume all sclerenchyma) are 80% dense and make up 40% of face
  $\sigma_{yf} = 1.44(0.8 \times 0.4 \times 100) + 1.53 = 47$ MPa

Iris Sandwich Analysis

Table 6.2 Beam bending results

Specimen	**1**	**2**	**3**	**4**
Measured beam stiffness, P/δ (N/mm)	0.66	0.54	0.41	0.25
Beam length, l (mm)	35	35	35	35
Face thickness, f (mm)	0.03	0.03	0.03	0.03
Maximum core thickness, c (mm)	4.63	3.31	2.49	1.51
Width, b (mm)	18	18	18	18
Flexural rigidity, D (Nm^2)	0.027	0.016	0.0096	0.0051
Bending compliance, $(\delta/P)_b$ (m/N)	5.29×10^{-4}	8.98×10^{-4}	1.49×10^{-3}	2.83×10^{-3}
Shear compliance, $(\delta/P)_s$ (m/N)	2.99×10^{-4}	3.83×10^{-4}	4.83×10^{-4}	6.3×10^{-4}
Calculated beam stiffness, P/δ (N/mm)	1.21	0.78	0.51	0.29
Calculated/measured beam stiffness	1.83	1.44	1.24	1.16

Core: $\tau_c^* =?$

- Literature $\quad \sigma^*_{\text{tenstion}} \approx 0.4$ MPa (parenchyma)
- Expect $\tau_c^* \sim 1/2\sigma^*_{\text{tenstion}} \sim 0.2$ MPa

- Calculate strength of iris leaf in wind — cantilever, uniformly distributed load ($B_3 = 2 \quad B_4 = 1$)
- Calculate loads at base of leaf (M_{max}): $t \sim 0.03$ mm $l \sim 600$ mm

face yielding: $\dfrac{P_{fy}}{b\,c} = B_3\,\sigma_{ys}\left(\dfrac{t}{l}\right) = (2)(47\text{ MPa})\left(\dfrac{0.03}{600}\right) = 4.7\text{ kPa}$

face wrinkling: $\dfrac{P_{fw}}{b\,c} = 0.57\,B_3\,E_f^{1/3}\,E_c^{*\,2/3}(t/l) = (0.57)(2)(8.2\times10^9)^{1/3}\,(4\times10^6)^{2/3}\left(\dfrac{0.03}{600}\right)$

core shear: $\dfrac{P_{cs}}{b\,c} = B_4\,\tau_c^* = (1)\,(0.2\text{MPa}) = 200\text{ kPa}$

- Expect leaf failure by face wrinkling

- Are iris leaves optimized?
  - $\delta_s/\delta_b = 0.22$-0.57 in specimens tested
  - in minimum weight design for given stiffness $\delta_s/\delta_b = 2$
  - but leaves have several functions beyond mechanical support:
    - photosynthesis requires large surface area
    - fluid transport
  - difficult to quantify relative importance of each function to plant
  - engineering optimization not possible

Additional examples of sandwich structures in nature:

- Marine "leaves" seaweed Durvillaea antarctica: fronds 12 m long
honeycomb core
- Bird skulls
  - if inner and outer face concentric — trabeculae oriented perpendicular to cortical shell
  - if inner and outer face not concentric — trabeculae foam-like
  - larger birds have multiple sandwiches
    - since size of trabeculae relatively constant, this may allow larger core thickness
  - owl skull — asymmetry — improves hearing

Comparison of optimized sandwich plate with solid plate of same stiffness

- Consider circular plate, radius R, simply supported ground circumference, subject to a uniformly distributed load q (N/m^2)
- Central plate deflection is ω
- If sandwich is optimized (based on analysis in book p.384)

$$\frac{\text{mass}}{\pi R^2} = 1.49 \left(\frac{q\,R}{\omega E_s}\right)^{3/5} \rho_s R \qquad \text{(foam core)}$$

- Equivalent solid plate:

$$\frac{\text{mass}}{\pi R^2} = 0.89 \left(\frac{q\,R}{\omega E_s}\right)^{1/5} \rho_s R$$

- Taking the ratio:

$$\frac{m_{\text{sandwich}}}{m_{\text{solid}}} = 1.67 \left(\frac{q\,R}{\omega E_s}\right)^{0.27}$$

- Consider bone sandwich, "foamed" trabecular core: R=100 mm, P=500 N, $\omega = 1$ mm
  $(= q\pi R^2)$ $\quad E_s = 18$ GPa

$$\frac{q\,R}{\omega\,E_s} = 10^{-4} \qquad \frac{m_{\text{sandwich}}}{m_{\text{solid}}} = 14\% \Rightarrow$$

optimized bone sandwich would be 14% weight of solid cortical panel

Additional examples (continued)

- Cuttlefish bone (not a fish — a mollusk; not bone — $CaCo_3$)
- Horseshoe crab shell
- Tortoise shell (Galapagos)

Durvillaea antarctica (New Zealand Seakelp)

Largest intertidal seaweed
Fronds up to 12m long
Fronds have gas-filled honeycomb-like core that provides buoyancy as well as flexural rigidity, maximizing surface area exposed to sunlight

http://en.wikipedia.org/wiki/File:Dried_bull_kelp_(Durvillaea_antarctica)_with_cross-section_showing_honeycomb_structure_IMG_102_1239.JPG

Bird Skulls

Images of bird skulls removed due to copyright restrictions. See Figure 6.7: Gibson, L. J., M. Ashby, et al. *Cellular Materials in Nature and Medicine*. Cambridge University Press, 2010. http://books.google.com/books?id=AKxiS4AKpyEC&pg=PA176

Courtesy of Alison Curtis. Used with permission.

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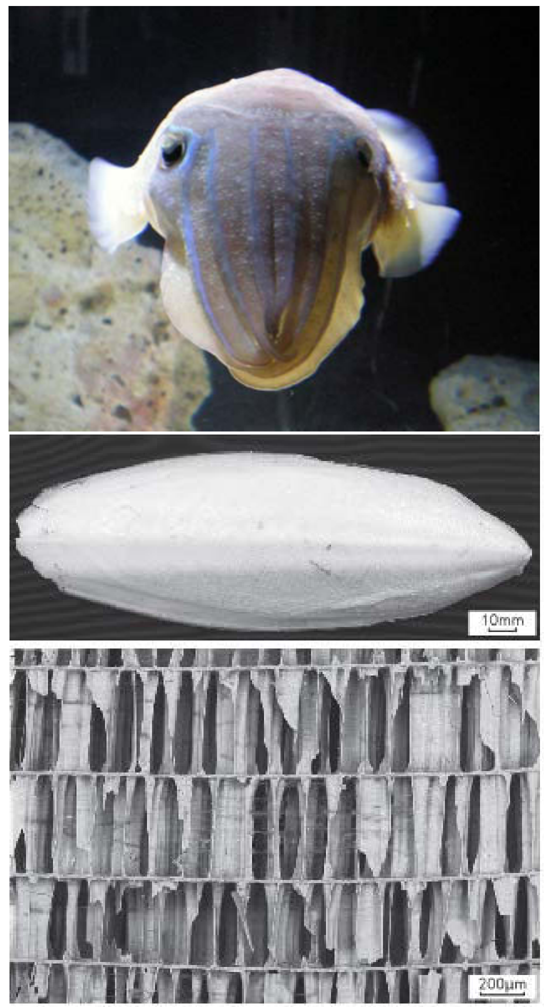
Photo of owl imprint in the snow removed due to copyright restrictions.

No footprints in the snow from mouse or vole; animal was under the snow
http://www.twincitiesnaturalist.com/2010/01/barred-owl-hunting-in-snow.html

Photo removed due to copyright restrictions. See Summit Post.
http://www.summitpost.org/disappearing-rabbit-trick/185785/c-186336

Rabbit tracks in snow
http://www.myconfinedspace.com/2006/12/21/owl-snowprint/

Cuttlefish bone
Mollusc shell ($CaCO_3$)



Horseshoe Crab Shell

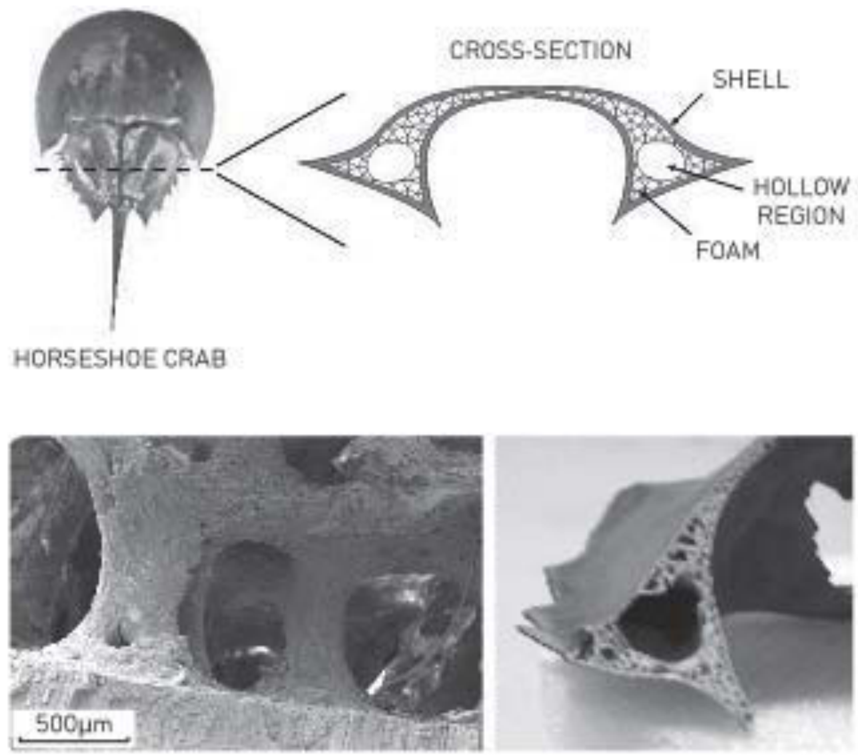


Figure 148: M. A. Meyers, P. -Y. Chen, et al. *Progress in Materials Science* 53 (2008): 1–206.

http://www.sciencedirect.com/science/article/pii/S0079642507000254

Meyers et al., 2008

Galapagos Tortoise Shell

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
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